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# PARTICLE MOTION WITH ROTATIONAL MOMENTUM IN KALUZA-KLEIN'S THEORY WITH MINIMALLY COUPLED GRAVITATIONAL AND ELECTROMAGNETIC FIELDS

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***Abstract* — In this work we investigate the geodetic motion with rotational momentum in the Kaluza Klein theory with minimally coupled gravitational and electromagnetic fields. We find that the orbits follow generalized Kepler ellipses, where these orbits undergo a rotation of the perihelion and the ellipses are disturbed by an additional wobbel motion in the radial direction. This disturbance is caused by a rotational potential that becomes gravitationally active. When the masses of the particles envolved is increased, this rotational potential can amplify to a potential barrier that devides the orbit into a confined inner orbit and an outer orbit. When we apply these findings to micro physics, the generalized Kepler ellipses describe the electro magnetic interaction force, whereas the cases with increased masses organically belong to the electro strong and electro weak interaction forces. In the case of the electro magnetic interaction force we can look for ideal circular orbits. We find that discrete orbits of such kind exist and that they coincide with Bohr's circular orbits in the atom. Quite naturally, from these findings a numerical procedure can be defined in order to determine the Planck constant numerically, which is the constant increment in rotational momentum between adjacent circular orbits. In order that such a constant increment exists, which guarantees the stability of the atom, none of the other natural constants can have a deviating value to the value taken from the measurements.**

***Index Terms* — particle motion, rotational momentum, gravitational and electromagnetic fields.**

## I. INTRODUCTION

In the work "Space travel of charged test particles hitting the singularity of a charged central gravitational center", by J.M.Schmidt, published in the International Journal of Technical Research and Applications (IJTRA), vol. 4, issue 6, November-December 2016, the Kaluza-Klein theory for minimally and non-minimally coupled gravitational and electromagnetic fields was worked out and solved for the spherically symmetric case. The corresponding geodetic equations for the particle motion were established for this spherically symmetric case and solved for the minimally coupled gravitational and electromagnetic fields in the case that the orbiting particle has no rotational momentum. Here, we want to investigate the particle motion in the case with spherical symmetric and minimally coupled gravitational and electromagnetic fields further, when the orbiting particle has a non vanishing rotational momentum. This motion is closely linked with Planck's constant , and we have to expand on this relation briefly.

## II. REMARKS ON PLANCK'S CONSTANT

Planck's constant h is considered as one of the most fundamental constants in nature, and the whole branch of quantum mechanics is built on it. The constant was first found by Max Planck at the beginning of the 20th century as the constant amount of energy per frequency photons carried away when they are emmitted by a radiating ideal black body. Later on, Niels Bohr speculated that electrons in an atom can only propagate along circular trajectories around the nucleus of the atom, where the rotational momentum of the electron is an integer multiple of the Planck constant, thus explaining the line spectrum of the hydrogen. Although later the atom was tackled as a system in which the electrons are statistically fluctuating entities that have to be described with a probability distribution function, the so-called wave function, it was still assumed that the energy per frequency is released in packages of size h, and is the energy operator in the equation that governs the wave function, which retrieves the energy from the wave function as an eigen value. As a consequence, the absolute squares of the wave functions in a hydrogen atom have still maxima at the locations of the Bohr orbits. This means that the whole edifice of quantum mechanics still rests on Bohr's fundamental assumptions with descrete orbits, determined by h. The mysterious thing is that in classical mechanics an electron can orbit a proton on a circular trajectory with any rotational momentum. So nobody knows where the restriction is coming from. Thus, h appears to be a constant that determines the

whole building of quantum mechanics, but has no correlations to any other mechanisms of which nature is constructed.

In this article we modify Bohr's approach by treating the orbiting electron-proton system as a general relativistic system and not as a system of classical mechanics. Since masses and charges are involved in such a system, the appropriate gravitational theory is the Kaluza-Klein theory, in which the gravitational and electromagnetic fields are minimally coupled (non-minimally coupled fields can play a role, but we can consider this influence as second order). In this theory, the trajectories of the electron are also in an ecliptic plane, and the motion of the electron is determined with an effective potential, which incorporates a gravitationally active rotational momentum of the orbiting electron. This gravitationally active rotational momentum is repulsive, if the electron approaches the proton, and attractive, when the electron leaves the proton. This has as a consequence that the trajectory of the electron around the proton has a wobble motion on top of it, which makes the trajectory to deviate from an ideal circular or elliptic orbit. Only in special balanced cases we get ideal circular orbits, also in the general relativistic theory. It can be shown that these ideal circular orbits of the relativistic theory coincide with the Bohr trajectories with a high degree of precision. Thus, the dynamics of the general relativistic system explain the restrictions on the trajectories of electrons orbiting a proton, which are needed to produce coherent emmission, and which are characterized with the Planck constant. Or in other words, the Planck constant is a consequence of the general relativistic laws. This result puts the edifice of quantum mechanics on solid ground. We not only understand where the Planck constant is coming from, we are also able to derive its numerical value from theory.

## III. THE TECHNICAL APPROACH

The technical approach to this problem is as follows. We first derive the effective or pseudo potential, which controls the general relativistic motion of electrons around protons. We then integrate the equation of motion in order to arrive at generalized Kepler ellipses, which can be expressed in terms of elliptic functions, and which incorporate the wobble motion I was talking about earlier. This derivation is a bit lengthy, but I chose a step by step approach so that anybody who follows that calculation does not stumble at a specific step, since he cannot find the link that leads to the following step. It is a good exercise for anybody who wishes to use elliptic integrals in his investigations. Moreover, having the shapes of the orbits in analytic form is as useful as having the Kepler ellipses in Kepler's time, since only then it was understood how our solar system really looks like. And these formula can be used in any further investigations of general relativistic motions of particles around central bodies.

The nice thing about the pseudo potential of a general relativistic particle orbiting a central particle is that it is a polynominal of fourth order in the inverse central distance coordinate. Thus, it has four roots, and the physical cases are those where either all roots are real or at least one pair of the four roots are real. Two neighbouring roots determine the aphelion and perihelon of an orbit. So if these two roots are close to each other or coincide, then the trajectory is close to or a circle. In order to find energy and rotational momentum parameters that belong to circular orbits, all we need to do is to insert these parameters into the pseudo potential and to look if two of the real roots of the corresponding polynominal of fourth order are close to each other or coincide. If the roots are not close to each other or coincide, we need to further fine-tune the energy and rotational momentum parameters as long until they do. Here, we use a shortcut of this method. Since we want to verify that the Bohr values for energy and rotational momentum of a Bohr trajectory also lead to a circular orbit in the general relativistic case, we take the Bohr values for energy and rotational momentum of the n-th Bohr trajectory as an estimate for the values for energy and rotational momentum in the general relativistic case and see that with these values the two roots of the pseudo potential are very close to each other. I have demonstrated this for the 1st, 2nd and 3rd of Bohr's orbits. This looks as if one takes information about the Planck constant, which flows into the estimates, in order to gain information about the Planck constant. So is here a flaw in our argumentation? No, it isn't, since the estimates are simple numerical numbers that could have been gained by any reasoning. The crucial thing is the selection criterion, if the roots coincide. This is an entirely numerical experiment independent from the Planck constant. What is lacking is that one should convince oneself that if one chooses energy and rotational momentum parameters deviating from the estimates that one then also obtains roots that are not so close to each other. This one can do extensively (and I have done that). Secondly, one has to do the verification for the 3rd, 4th, of Bohr's orbits. I have done this, too, but haven't incorporated it in this manuscript. This discussion would have blown up an already extensive manuscript. Yet, I encourage everybody to launch his own investigations. What one finds is that the coincidence of the roots becomes less prominent the higher the quantum number n is. This is no surprise, since in the end our theory is just a model that has assumptions in it (e.g., perfect spherical symmetry, linear coupling between the fields, etc.).

There is a further implication of this study. The pseudo potential depends on the masses and the charges of the electron and the proton, and the coupling constants of the fields, next to the energy and the rotational momentum parameter. If one changes one of the mass or charge parameters of the electron or proton, or one changes one of the coupling constants of the fields, then the corresponding pseudo potential determines circular orbits that do not have a ladder spectrum of equidistant rotational momentum values. An atom with such dimensions would not be able to emit coherent emmission, and was therefore virtually undetectable or non-existing. Only a system, where the mass and charge values for the electron and the proton and the coupling constants of the fields have the measured values realized in nature, develops a ladder spectrum with equidistant values of the rotational momentum of its circular trajectories, and is visible and stable. So the laws that the proton mass is 1836 times the electron mass and the charge of the proton is opposite to the charge of the electron are consequences of the general relativistic laws, too.

## IV. MOTION WITH ROTATIONAL MOMENTUM

### A. Introductory Remarks

Kaluza-Klein's theory with minimally coupled fields is a theory in which the gravitational field is coupled linearly with the electromagnetic field. The formulation of this theory follows the concept of Einstein to describe forces in a geometrical manner via metric fields that vary depending on the relative motion of the observer and the object. Despite the possibility of a nonlinear coupling between the gravitational and the electromagnetic field, which can be described in a Kaluza-Klein theory with nonlinear coupling, the Kaluza-Klein theory with minimal coupling is the simplest theory that unifies gravitation and electromagnetism. Because of its relative simplicity, the motion of charged particles can be described in great analytic detail in this theory. Like Newton, who calculated the motion of the planets exactly, which gave us deep insight in celestrial dynamics, such analytic studies can give us insight in general relativistic effects that alter the classical dynamics of particles, when the gravitational forces become quite strong. An example for such a system is an elementary particle that propagates in the vicinity of the center of the nucleus of an atom, since both, the gravitational as well as the electromagnetic field have a singularity at that center.

This work is intended to derive the equivalent of Kepler's ellipses in Kaluza-Klein's theory with minimal coupling. Such generalized ellipses can be described in terms of elliptical functions. We will give this derivation in great detail, since it is a good exercise to improve our integration capabilities. Furthermore, this approach will show that the investigation can easily be extended in different directions. Thus, this work is also intended to encourage such further investigations. The work is not intended to be complete.

### B. The Equations of Motion.

The geodetic eqations for the motion of a charged particle in Einstein-Maxwell's theory for the spherically symmetric case read [1]:

$$e^{\nu}\cdot c\dot{t}+qA_0=\beta \tag{1}$$

$$\dot{\phi}=\frac{\alpha}{r^2} \tag{2}$$

$$\theta=\frac{\pi}{2} \tag{3}$$

$$-c^2=-e^{\nu}\cdot c^2[\dot{t}]^2+e^{\lambda}[\dot{r}]^2+r^2\left([\dot{\theta}]^2+\sin^2\theta[\dot{\phi}]^2\right) \tag{4}$$

Here, $r$, $\theta$ and $\phi$ are sperical coordinates and $t$ is the coordinate time. The dot denotes the differentiation with respect to the "Eigen"-time $\tau$ of the propagating particle, where we use the convention $ds^2=-c^2d\tau^2$ between the line-element $ds$ and the "Eigen"-time, and $c$ is the light-velocity. Thus, equation (4) is the definition of the line-element, which turns out to be one of the geodetic equations. The functions $e^{\nu}$ and $e^{\lambda}$ are the metric fields in a rotational symmetric stationary metric. For the Reißner-Nordström solution we have $e^{\nu}=e^{-\lambda}=1-\frac{2m}{r}+\frac{\kappa^2k^2}{2r^2}$, where $2m$ is Schwarzschild's radius with $m=\frac{MG_N}{c^2}$, $M$ is the mass of the central body, $G_N$ is Newton's gravitational constant, $\kappa^2=\frac{8\pi G_N}{c^4}$ is the coupling-constant between the gravitational and the electromagnetic field, $k=\frac{Q}{4\pi\varepsilon_0}$, $Q$ is the charge of the central body, and $\varepsilon_0$ is the dielectric constant. Equation (1) is a generalized "energy"-law, where $q=\frac{e}{m_ec}$, $e$ and $m_e$ are the charge and the mass of the orbiting particle, $\beta=\frac{E}{m_ec}$ is the total energy of the orbiting particle, normalized to $m_ec$, and $A_0=\frac{k}{r}$ is the Coulomb-potential of the central body, which is the zero-component of the vector-potential. Equation (2) is the conservation-law of the rotational momentum like in the classical case, where $\alpha=\frac{l}{m_e}$ is the rotational momentum that is normalized to the mass of the orbiting particle. Equation (3) fixes the $\theta$-coordinate, e.g. the motion of the particle is within the ecliptic plane.

Since there is a radial varying metric component $e^{\nu}$ in the energy-equation, the rotational momentum of the orbiting particle is "out of phase" with the motion of the orbit, if we compare the motion with the classical motion. This leads to an additional potential-well for the orbiting particle, when it approaches the gravitational center, which distorts the classical motion. This effect can become so prominent, that a further approach of the center is completely violated. In that case, the well is also repulsive for a particle within the well that can not escape the vicinity of the nucleus. This effect nicely corresponds to the experimental effect of "confinement" of elementary particles that are influenced by the

"strong interaction". Apparently, the "strong interaction" is a general relativistic effect for closely orbiting elementary particles with

strongly interacting gravitational and electromagnetic fields.

For an elementary particle like the proton, the Reißner-Nordström metric has no event horizons. An orbiting particle can get as close to the gravitational center as it is

possible. In extreme cases, e.g. if the rotational momentum is zero or close to zero, the orbiting particle can move straightforwardly into the singularity at the gravitational center. Yet, since the equations of motion are smooth, the particle is not repulsed at the central singularity. Instead, it tunnels right through the singularity. In mathematical terms, it moves from the space with positive radial coordinate $r$ into the space with **negative** radial coordinate $r$. As we see from the definition of the Coulomb-potential in the equation of motion which is proportional to $1/r$ and proportional to $e$, the charge of the particle, a switch to negative $r$ corresponds to a switch to a particle with positive $r$-coordinate, but with the **opposite** charge. I.e. the elementary particle transforms to its anti-particle, when it crosses the central singularity. If we look at the energy-equation (1), we see, that a change of the sign of the term $qA_0$ has a change of sign of the term $\dot{t}$ as its consequence, since the term $1/r^2$ in $e^{\nu}$ is dominant close to the singularity, which does not change its sign, and $\beta$ is a constant. This means that the particle propagates **backwards in time**, as soon as it transforms to its anti-particle. Since all orbits are periodic, the anti-particle returns to the singularity after it has traveled backwards in time for a while. When it hits the singularity again, it transforms to the former elementary particle and leaves the singularity as if it has been repulsed. Yet, from the point on that it has become the former elementary particle again, it propagates forwards in time again. It reappears **before** it has actually hit the singularity in the process beforehand. The time-span between the moment of reappearance and the time when the particle hits the singularity first is exactly the absolute value of the time-span that it propagated backwards in time as an anti-particle. Thus, the falling particle and the reappeared particle are also spatially separated. This process can also be described as if a particle interacts with a particle anti-particle pair that it hits out of "Dirac's ocean". Yet, in the general relativistic picture the three particles are related by general relativistic laws. The replacement of the same particle in time and space due to the effect that has been described could be the explanation for the fact that we actually see the fluctuation of elementary particles in microphysics that leads to the quantum-mechanical behaviour of nature. The linkage of these fluctuations with general relativistic propagation-laws could be the reason for the fact that these observed fluctuations follow the law of Heisenberg's uncertainty equation. We have expanded on these facts in Schmidt (2016).

a) The Pseudo-potential for the Motion of the Particle

We take $\dot{t}$ from equation (1), $\dot{\phi}$ from equation (2) and $\theta$ form equation (3) and substitute them into equation (4). This yields

$$-c^2 = -e^{-\nu}[\beta - qA_0]^2 + e^{\lambda}[\dot{r}]^2 + \frac{\alpha^2}{r^2}. \tag{5}$$

We define

$$u = \frac{1}{r} \tag{6}$$

and get with the help of (2):

$$\dot{r} = \frac{dr}{d\tau} = \alpha u^2 \frac{d}{d\phi}\left(\frac{1}{u}\right) = -\alpha \frac{du}{d\phi}. \tag{7}$$

This leads to

$$\left[\frac{du}{d\phi}\right]^2 + \left[\left(u^2 + \frac{c^2}{\alpha^2}\right)e^{-\lambda(u)} - \right.$$
$$\left. - \frac{1}{\alpha^2}[\beta - qA_0(u)]^2 e^{-(\nu(u)+\lambda(u))}\right] = 0.$$

The term

$$V_{pseudo}(u,\beta,\alpha) = \left(u^2 + \frac{c^2}{\alpha^2}\right)e^{-\lambda(u)} -$$
$$- \frac{1}{\alpha^2}[\beta - qA_0(u)]^2 e^{-(\nu(u)+\lambda(u))}$$

in this equation can be looked at as an energy- and rotational momentum depending "pseudo-potential" for a motion of a particle at coordinate $u$ with the parameter of the motion $\phi$. For the Reißner-Nordström solution and the Coulomb-potential we get

$$V_{pseudo}(u,\beta,\alpha) = a_1(u^4 + \bar{a}u^3 + \bar{b}u^2 + \bar{c}u + \bar{d}) \tag{10}$$

with

$$a_1 = \frac{\kappa^2 k^2}{2}$$
$$\bar{a} = -\frac{2m}{a_1}$$
$$\bar{b} = \frac{\alpha^2 + a_1 c^2 - q^2 k^2}{a_1 \alpha^2}$$
$$\bar{c} = \frac{2[\beta q k - mc^2]}{a_1 \alpha^2}$$
$$\bar{d} = \frac{c^2 - \beta^2}{a_1 \alpha^2},$$

i.e. $V_{pseudo}(u,\beta,\alpha)$ is a polynomial of degree four in $u$. The general solution of

$$\left[\frac{du}{d\phi}\right]^2 + V_{pseudo}(u,\beta,\alpha) = 0 \tag{16}$$

is

$$\int^{u} \frac{du'}{\sqrt{-\frac{1}{a_1}V_{pseudo}(u',\beta,\alpha)}} = \sqrt{a_1}(\phi - \phi_0). \tag{17}$$

Here, the integration constant $\phi_0$ has no physical meaning. That constant corresponds to an arbibrary rotation within the ecliptic plane. The difficult bit is to proceed with the integration further. We will address this problem in the next two sections.

b) Integration in the Case of a Parabola-like Pseudo-potential

c) *We look at the case that the pseudo-potential has the form*

$$\frac{1}{a_1}V_{pseudo}(u,\beta,\alpha) = (u-u_0)(u-u_1)\left[(u-\mu)^2+\nu^2\right],$$

i.e. two of the roots of the polynomial are complex conjugates $u_{2/3} = \mu \pm i\nu$ and the motion is restricted to $u \in [u_0; u_1]$.

We look at the function

$$u' = \frac{(u_1B + Au_0) - (u_1B - Au_0)\sqrt{1-x'^2}}{(B+A) - (B-A)\sqrt{1-x'^2}}, \qquad (19)$$

where

$$A^2 = (\mu-u_1)^2 + \nu^2$$
$$B^2 = (\mu-u_0)^2 + \nu^2$$
$$K^2 = \frac{(u_1-u_0)^2 - (A-B)^2}{4AB}.$$

$$\Rightarrow u'[(B+A) - (B-A)\sqrt{1-x'^2}] =$$
$$(u_1B + Au_0) - (u_1B - Au_0)\sqrt{1-x'^2}$$
$$\Rightarrow (B+A)u' - (u_1B + Au_0) =$$

$$(B-A)u'\sqrt{1-x'^2} - (u_1B - Au_0)\sqrt{1-x'^2}$$
$$= [(B-A)u' - (u_1B - Au_0)]\sqrt{1-x'^2}$$

$$\Leftrightarrow \sqrt{1-x'^2} = \frac{(B+A)u' - (u_1B + Au_0)}{[(B-A)u' - (u_1B - Au_0)]}$$

$$\Leftrightarrow 1-x'^2 = \left[\frac{(B+A)u' - (u_1B + Au_0)}{(B-A)u' - (u_1B - Au_0)}\right]^2$$

$$\Leftrightarrow x'^2 = 1 - \left[\frac{(B+A)u' - (u_1B + Au_0)}{(B-A)u' - (u_1B - Au_0)}\right]^2$$

$$= \frac{(-4)AB(u'-u_1)(u'-u_0)}{[(B-A)u' - (u_1B - Au_0)]^2} \Rightarrow x' = -2\sqrt{AB}\sqrt{(-1)}\cdot$$
$$\cdot \frac{\sqrt{u'-u_1}\sqrt{u'-u_0}}{[(B-A)u' - (u_1B - Au_0)]},$$

where we have chosen the negative sign in order to secure that a decending $x'$ belongs to an ascending $u'$.

$$\Rightarrow 2x'dx' = (-4)AB([(u'-u_1)+(u'-u_0)]\cdot$$
$$\cdot[(B-A)u' - (u_1B - Au_0)] -$$
$$-2(B-A)(u'-u_1)(u'-u_0))$$
$$/[(B-A)u' - (u_1B - Au_0)]^3 \cdot du'$$

$$\Rightarrow \frac{dx'}{\sqrt{AB}} = \frac{1}{\sqrt{(-1)}\sqrt{u'-u_1}\sqrt{u'-u_0}}\cdot$$
$$\cdot([(u'-u_1)+(u'-u_0)]\cdot$$
$$\cdot[(B-A)u' - (u_1B - Au_0)] -$$
$$-2(B-A)(u'-u_1)(u'-u_0))$$
$$/[(B-A)u' - (u_1B - Au_0)]^2 \cdot du'$$

(18) (20) (21) (22)

In this expression we have

$$[(u'-u_1)+(u'-u_0)]\cdot$$
$$\cdot[(B-A)u' - (u_1B - Au_0)] -$$
$$-2(B-A)(u'-u_1)(u'-u_0)$$

$$= (u_1-u_0)[(-1)(B+A)u' +$$
$$+(u_1B + Au_0)]$$

Thus, it is

$$\frac{dx'}{\sqrt{AB}} = \frac{1}{\sqrt{(-1)}\sqrt{u'-u_1}\sqrt{u'-u_0}}\cdot$$
$$\cdot\frac{(u_1-u_0)[(-1)(B+A)u' + (u_1B + Au_0)]}{[(B-A)u' - (u_1B - Au_0)]^2}\cdot$$
$$\cdot du'$$

On the other hand we have

$$(1-x'^2)(1-K^2x'^2) =$$

$$= ([(B-A)u' - (u_1B - Au_0)]^2 +$$
$$+4AB(u'-u_1)(u'-u_0))$$
$$/[(B-A)u' - (u_1B - Au_0)]^2 \cdot$$
$$\cdot([(B-A)u' - (u_1B - Au_0)]^2 +$$

$$+[(u_1-u_0)^2-(A-B)^2](u'-u_1)(u'-u_0))$$
$$/[(B-A)u'-(u_1B-Au_0)]^2$$

In this expression it is

$$[(B-A)u'-(u_1B-Au_0)]^2+$$
$$+4AB(u'-u_1)(u'-u_0)$$

$$=[(-1)(A+B)u'+(u_1B+Au_0)]^2$$

and

$$[(B-A)u'-(u_1B-Au_0)]^2+$$

$$+[(u_1-u_0)^2-(A-B)^2](u'-u_1)(u'-u_0)$$

$$=(u_1-u_0)^2u'^2-$$
$$-[B^2-A^2+u_1^2-u_0^2](u_1-u_0)u'+$$
$$+(u_1-u_0)[u_1(B^2-u_0^2)-$$
$$-u_0(A^2-u_1^2)]$$

It is

$$[B^2-A^2+u_1^2-u_0^2]=(\mu-u_0)^2+\nu^2-(\mu-u_1)^2-$$
$$-\nu^2+u_1^2-u_0^2=2\mu(u_1-u_0)$$

and

$$[u_1(B^2-u_0^2)-u_0(A^2-u_1^2)]=(u_1-u_0)(\mu^2+\nu^2)$$

Thus

$$[(B-A)u'-(u_1B-Au_0)]^2+$$
$$+[(u_1-u_0)^2-(A-B)^2](u'-u_1)(u'-u_0)$$
$$=(u_1-u_0)^2u'^2--2\mu(u_1-u_0)(u_1-u_0)u'+$$
$$+(u_1-u_0)(u_1-u_0)(\mu^2+\nu^2)$$
$$=(u_1-u_0)^2[u'^2-2\mu u'+\mu^2+\nu^2]$$
$$=(u_1-u_0)^2[(u'-\mu)^2+\nu^2]$$

We can substitute the last two simplified polynomials into the original expression. This yields

$$(1-x'^2)(1-K^2x'^2)=$$
$$=\frac{[(-1)(A+B)u'+(u_1B+Au_0)]^2}{[(B-A)u'-(u_1B-Au_0)]^2}\cdot$$
$$\cdot\frac{(u_1-u_0)^2[(u'-\mu)^2+\nu^2]}{[(B-A)u'-(u_1B-Au_0)]^2}.$$

$$\Rightarrow\frac{1}{\sqrt{(1-x'^2)(1-K^2x'^2)}}=$$
$$=[(B-A)u'-(u_1B-Au_0)]^2$$
$$/[(-1)(A+B)u'+(u_1B+Au_0)]$$
$$/(u_1-u_0)/\sqrt{(u'-\mu)^2+\nu^2}$$

If we combine this with the expression for the differential $dx'$, we get

$$\frac{dx'}{\sqrt{AB}}\cdot\frac{1}{\sqrt{(1-x'^2)(1-K^2x'^2)}}=$$
$$=\frac{1}{\sqrt{(-1)}\sqrt{u'-u_1}\sqrt{u'-u_0}}\cdot$$
$$\cdot(u_1-u_0)$$
$$\cdot[(-1)(B+A)u'+(u_1B+Au_0)]$$
$$/[(B-A)u'-(u_1B-Au_0)]^2$$
$$\cdot du'\cdot$$
$$\cdot[(B-A)u'-(u_1B-Au_0)]^2$$
$$/[(-1)(A+B)u'+(u_1B+Au_0)]$$
$$/(u_1-u_0)$$
$$/\sqrt{(u'-\mu)^2+\nu^2}$$

$$\Leftrightarrow\frac{1}{\sqrt{AB}}\cdot\frac{dx'}{\sqrt{(1-x'^2)(1-K^2x'^2)}}=$$

$$=\frac{du'}{\sqrt{(-1)(u'-u_1)(u'-u_0)[(u'-\mu)^2+\nu^2]}}$$

We can integrate this equation:

$$\Rightarrow\frac{1}{\sqrt{AB}}\cdot\int^{x}\frac{dx'}{\sqrt{(1-x'^2)(1-K^2x'^2)}}=$$

$$=\int^{u}\frac{du'}{\sqrt{(-1)(u'-u_1)(u'-u_0)[(u'-\mu)^2+\nu^2]}}$$

$$=\int^{u}\frac{du'}{\sqrt{-\frac{1}{a_1}V_{pseudo}(u',\beta,\alpha)}}=\sqrt{a_1}(\phi-\phi_0),$$

where we used equation (17). On the other hand it is

$$\int^{x}\frac{dx'}{\sqrt{(1-x'^2)(1-K^2x'^2)}}=JacobiSN^{-1}(x,K),$$

where $JacobiSN^{-1}(x,K)$ is the inverse function of the elliptical Jacobi function $JacobiSN(x,K)$.

$$\Rightarrow \frac{1}{\sqrt{AB}} \cdot JacobiSN^{-1}(x,K) = \sqrt{a_1}(\phi - \phi_0)$$

$$\Leftrightarrow JacobiSN^{-1}(x,K) = \sqrt{a_1 AB}(\phi - \phi_0)$$

$$\Leftrightarrow x = JacobiSN(\sqrt{a_1 AB}(\phi - \phi_0), K),$$

when we apply the operation $JacobiSN(*,K)$ on the second equation. Between the elliptical function $JacobiSN(y,K)$ and the elliptical function $JacobiCN(y,K)$ there is the relation

$$JacobiCN(y,K) = \sqrt{1 - JacobiSN^2(y,K)}. \quad (23)$$

Thus

$$\sqrt{1-x^2} = \sqrt{1 - JacobiSN^2(\sqrt{a_1 AB}(\phi - \phi_0), K)}$$
$$= JacobiCN(\sqrt{a_1 AB}(\phi - \phi_0), K)$$

Now, we can use our original relation (19), and with $u = 1/r$ we get

$$r(\phi) = ((B+A) - (B-A) \cdot$$
$$\cdot JacobiCN(\sqrt{a_1 AB}(\phi - \phi_0), K))$$
$$/((u_1 B + A u_0) - (u_1 B - A u_0) \cdot$$
$$\cdot JacobiCN(\sqrt{a_1 AB}(\phi - \phi_0), K))$$

Equation (24) is the exact solution of the trajectory of the particle in the ecliptic $r$-$\phi$-plane in terms of elliptic functions. We realize that the Jacobi function $JacobiCN(*,K)$ is a function close to the trigonometric cosine function, although the Jacobi function has a modified periodicity, with accounts for the rotation of the perihelion for that general relativistic orbit. Thus, equation (24) is quite close to a Kepler ellipse, which has the form $P/(1-\varepsilon\cos(\phi))$, where $\varepsilon$ is the excentricity and $P$ a trajectory parameter. The difference is that in equation (24) $P$ is also a periodic function of the Jacobi function. This generates the wobble motion of the orbiting particle on top of the motion along the ellipse. This wobble motion is suppressed only, when $(B-A)$ tends to zero. In this case $u_0$ and $u_1$ coincide and we have a perfect circular motion. We will show that these perfect circular motions are realized in the hydrogen atom and are identical with Bohr's orbits.

d) Integration in the Case of a Potential Well that Separates an Outer Orbit from a Confined Inner Orbit

In this case the pseudo-potential can be written as

$$\frac{1}{a_1} V_{pseudo}(u,\beta,\alpha) = (u-u_3)(u-u_2)(u-u_1)(u-u_0).$$

In the case that the roots of $V_{pseudo}(u,\beta,\alpha)$ are real and ordered, i.e. $u_0 < u_1 < u_2 < u_3$, the orbit $u \in [u_0; u_1]$ belongs to the outer orbit and the orbit $u \in [u_2; u_3]$ belongs to the confined inner orbit (recall that we have $u = 1/r$, i.e. a large border-value $u_i$ belongs to an actual small radial distance $r_i$ and vice versa). Yet, we note that the calculation that follows is general and doesn't depend on the requirement that the roots are real or ordered. Thus, we can relabel the roots at the end of the calculation as we like in our solution. This can be used to obtain the solution for the confined inner orbit, if we have derived the solution for the outer orbit.

We define the modulus

$$K = \sqrt{\frac{(u_3-u_2)(u_1-u_0)}{(u_3-u_1)(u_2-u_0)}} \quad (26)$$

and look at the function

$$u' = \frac{u_0(u_3-u_1) + u_3(u_1-u_0)x'^2}{(u_3-u_1) + (u_1-u_0)x'^2} \quad (27)$$

From this we get

$$u_0(u_3-u_1) + u_3(u_1-u_0)x'^2 =$$
$$u'[(u_3-u_1) + (u_1-u_0)x'^2]$$
$$\Leftrightarrow u_0(u_3-u_1) - u'(u_3-u_1) =$$
$$u'(u_1-u_0)x'^2 - u_3(u_1-u_0)x'^2 \quad (24)$$
$$\Leftrightarrow (u_0-u')(u_3-u_1) =$$
$$(u'-u_3)(u_1-u_0)x'^2$$
$$\Leftrightarrow x'^2 = (-1)\frac{(u_3-u_1)(u'-u_0)}{(u_1-u_0)(u'-u_3)}$$

Thus

$$x' = \sqrt{(-1)\frac{(u_3-u_1)(u'-u_0)}{(u_1-u_0)(u'-u_3)}} \quad (28)$$

and

$$2x'dx' = (-1)((u_3-u_1)(u_1-u_0)(u'-u_3) -$$
$$-(u_3-u_1)(u'-u_0)(u_1-u_0)) / [(u_1-u_0)(u'-u_3)]^2 \cdot du'$$

$$= \frac{(u_3-u_1)(u_3-u_0)}{(u_1-u_0)} \cdot \frac{du'}{(u'-u_3)^2}$$

It follows

$$2\sqrt{(-1)\frac{(u_3-u_1)(u'-u_0)}{(u_1-u_0)(u'-u_3)}}dx' =$$
$$\frac{(u_3-u_1)(u_3-u_0)}{(u_1-u_0)} \cdot \frac{du'}{(u'-u_3)^2} \quad (25)$$

$$\Leftrightarrow 2dx' = \frac{\sqrt{u_3-u_1}(\sqrt{u_3-u_0})^2}{\sqrt{(-1)}\sqrt{u_1-u_0}}\cdot$$
$$\cdot\frac{du'}{\sqrt{u'-u_0}(\sqrt{u'-u_3})^3}$$

On the other hand we have

$$(1-x'^2)(1-K^2x'^2) = \div[(u_1-u_0)(u_2-u_0)(u'-u_3)^2 +$$
$$+[(u_3-u_1)(u_2-u_0)+(u_3-u_2)(u_1-u_0)]\cdot$$
$$\cdot(u'-u_0)(u'-u_3)+(u_3-u_1)(u_3-u_2)(u'-u_0)^2]\cdot$$
$$\cdot\frac{1}{(u_1-u_0)(u_2-u_0)(u'-u_3)^2}$$

For the numerator in this expression we get

$$(u_1-u_0)(u_2-u_0)(u'-u_3)^2 +$$
$$+[(u_3-u_1)(u_2-u_0)+(u_3-u_2)(u_1-u_0)]\cdot$$
$$\cdot(u'-u_0)(u'-u_3)+(u_3-u_1)(u_3-u_2)(u'-u_0)^2 =$$
$$=(u_3-u_0)^2(u'-u_1)(u'-u_2)$$

Thus, we have

$$(1-x'^2)(1-K^2x'^2) = \frac{(u_3-u_0)^2(u'-u_1)(u'-u_2)}{(u_1-u_0)(u_2-u_0)(u'-u_3)^2}$$

$$\Leftrightarrow \frac{1}{\sqrt{(1-x'^2)(1-K^2x'^2)}} =$$
$$\frac{\sqrt{u_1-u_0}\sqrt{u_2-u_0}(\sqrt{u'-u_3})^2}{(\sqrt{u_3-u_0})^2\sqrt{u'-u_1}\sqrt{u'-u_2}}$$

If we multiply this equation with equation (29), we get

$$\frac{2dx'}{\sqrt{(1-x'^2)(1-K^2x'^2)}} =$$
$$=\frac{\frac{1}{\sqrt{(-1)}}\sqrt{u_3-u_1}\sqrt{u_2-u_0}du'}{\sqrt{u'-u_0}\sqrt{u'-u_1}\sqrt{u'-u_2}\sqrt{u'-u_3}}$$

$$\Leftrightarrow \frac{2}{\sqrt{(u_3-u_1)(u_2-u_0)}}\cdot$$
$$\cdot\frac{dx'}{\sqrt{(1-x'^2)(1-K^2x'^2)}} =$$

$$\frac{du'}{\sqrt{(-1)(u'-u_0)(u'-u_1)(u'-u_2)(u'-u_3)}}$$

Now, we can integrate equation (30). This yields

$$\frac{2}{\sqrt{(u_3-u_1)(u_2-u_0)}}\cdot\int_x\frac{dx'}{\sqrt{(1-x'^2)(1-K^2x'^2)}} =$$
(29)
$$=\int_u\frac{du'}{\sqrt{(-1)(u'-u_0)(u'-u_1)(u'-u_2)(u'-u_3)}}$$
$$=\int_u\frac{du'}{\sqrt{-\frac{1}{a_1}V_{pseudo}(u',\beta,\alpha)}} = \sqrt{a_1}(\phi-\phi_0),$$

according to equation (17). On the other hand we have

$$\frac{2}{\sqrt{(u_3-u_1)(u_2-u_0)}}\cdot\int_x\frac{dx'}{\sqrt{(1-x'^2)(1-K^2x'^2)}} =$$
$$\frac{2}{\sqrt{(u_3-u_1)(u_2-u_0)}}\cdot$$
$$\cdot JacobiSN^{-1}(x,K)$$
$$\Rightarrow \frac{2}{\sqrt{(u_3-u_1)(u_2-u_0)}}\cdot$$
$$\cdot JacobiSN^{-1}(x,K) = \sqrt{a_1}(\phi-\phi_0)$$

$$\Leftrightarrow JacobiSN^{-1}(x,K) = \sqrt{a_1(u_3-u_1)(u_2-u_0)}\cdot$$
$$\cdot\frac{(\phi-\phi_0)}{2}$$

$$\Rightarrow x = JacobiSN(\sqrt{a_1(u_3-u_1)(u_2-u_0)}\cdot$$
$$\cdot\frac{(\phi-\phi_0)}{2},K)$$

Now, we look at our original function (27), and with $u = 1/r$, we have

$$r(\phi) = ((u_3-u_1)+(u_1-u_0)\cdot$$
$$\cdot JacobiSN^2(\sqrt{a_1(u_3-u_1)(u_2-u_0)}\cdot$$
$$\cdot\frac{(\phi-\phi_0)}{2},K))\ /(u_0(u_3-u_1)+u_3(u_1-u_0)\cdot$$
$$\cdot JacobiSN^2(\sqrt{a_1(u_3-u_1)(u_2-u_0)}\cdot$$
$$\cdot\frac{(\phi-\phi_0)}{2},K))$$

The expression (31) is the exact trajectory of the particle in the ecliptic $r$-$\phi$-plane in the case of the motion with a repulsive potential well in terms of elliptic functions. Since

(30)

$JacobiSN(y,K) \in [0;1]$, (31) describes the motion for $r \in [1/u_1;1/u_0]$, i.e. it is the expression for the outer orbit.

In order to get the solution for the inner confined orbit, we need to exchange the two potential hills, where the motion is possible, i.e. we need to exchange $u_0 \leftrightarrow u_2$ and $u_1 \leftrightarrow u_3$. This yields

$$r(\phi) = ((u_3 - u_2)\cdot$$
$$\cdot JacobiSN^2(\sqrt{a_1(u_3-u_1)(u_2-u_0)}\cdot$$
$$\cdot\frac{(\phi-\phi_0)}{2}, K) - \cdot(u_3-u_1)) \,/(u_1(u_3-u_2)\cdot$$
$$\cdot JacobiSN^2(\sqrt{a_1(u_3-u_1)(u_2-u_0)}\cdot$$
$$\cdot\frac{(\phi-\phi_0)}{2}, K) - \cdot u_2(u_3-u_1))$$

as exact solution for the inner confined orbit. The modulus $K$ remains unchanged by this exchange-operation. It is clear that the inner confined orbit describes the electro-strong interaction between quarks. We come in this regime of the interaction, when we moderately diminish the charge of the particles, and increase the masses significantly. The potential well between the inner and outer orbit is a consequence of increasing the general relativistic effects. The particle on the inner orbit of another particle can have a third particle orbiting on its own inner orbit. Finally, the third particle can have the first particle on its inner orbit. Thus, we see that naturally three particles cluster together to form an entity. These are the nucleons that consist of three quarks each. Since each of the quarks in a nucleon has an outer orbit, too, another nucleon can be associated by saturating those bounds. This is clearly the electro-weak interaction that makes nucleons cluster together in atoms.

e) Investigation of Particle Motion at Bohr's Ground Level in the Hydrogen Atom with low Angular Momentum

In this section we will apply our findings about the general relativistic motion of a particle around a central particle to an electron that orbits a central proton. We will see that we can explain a number of properties of the hydrogen atom like the occurrance of Planck's constant and its multiples as the angular momentum of electrons on most stable orbits.

As basic means for this numerical investigation, the following REDUCE computer algebra program can be used. I will give detailed comments on that program after its listing in the text.

```
on rounded;
precision 20;
G_N := 6.672e-11;
c := 2.997925e8;
kappa_2 := 8 * pi * G_N / c**4;
e_el := 1.6022e-19;
Q_large := e_el;
epsilon_0 := 8.8542e-12;
k := Q_large / ( 4 * pi * epsilon_0 );
a_1 := kappa_2 * k**2 / 2;
M_N := 1.6726e-27;
m := M_N * G_N / c**2;
m_e := 9.1095e-31;
q := - e_el / ( m_e * c );
h := 6.6262e-34;
n := 1;
r := epsilon_0 * h**2 /
( pi * e_el**2 * m_e ) * n**2;
v := e_el**2 / ( 2 * epsilon_0 * h )
* 1 / n;
fac_1 := 1;
alpha := r * v /
sqrt( 1 - ( v / c )**2 ) * fac_1;
fac_2 := 1;
beta := ( m_e * c**2 /
sqrt( 1 - ( v / c )**2 ) -
e_el * Q_large /
( 4 * pi * epsilon_0 ) / r ) /
( m_e * c ) * fac_2;
a_bar := - 2 * m / a_1;
b_bar := ( alpha**2 + a_1 * c**2 -
q**2 * k**2 ) / ( a_1 * alpha**2 );
c_bar := 2 * ( beta * q * k -
m * c**2 ) / ( a_1 * alpha**2 );
d_bar := ( c**2 - beta**2 ) /
( a_1 * alpha**2 );
poly := u**4 + a_bar * u**3 +
b_bar * u**2 + c_bar * u + d_bar;
roots(poly);
u0 := rhs(third(roots(poly)));
u1 := rhs(first(reverse(roots(poly))));
u2 := rhs(first(roots(poly)));
u3 := rhs(second(roots(poly)));
mu:=(u2+u3)/2;
nu:=(u2-u3)*(-i)/2;
end;
```

(32)

The structure of the REDUCE program is rather simple. We first calculate the coefficients of $V_{pseudo}(u,\beta,\alpha)$ $a_1$, $\bar{a}$, $\bar{b}$, $\bar{c}$ and $\bar{d}$ as we have defined them in the theory section for the masses and charges of an electron and a proton. Since we perform a numerical study, we need to insert the measured values for the other natural constants, too. A little bit tricky is the estimate of the energy and the angular momentum parameters $\beta$ and $\alpha$. For $\beta$ we set

$$\beta \approx \frac{1}{m_e c}\left(\frac{m_e c^2}{\sqrt{1-(\frac{v}{c})^2}} - \frac{eQ}{4\pi\varepsilon_0}\frac{1}{r}\right). \qquad (33)$$

This means that we estimate the energy of the orbiting electron by the sum of its special relativistic kinetic energy, which is its dynamical mass $m_e c^2/\sqrt{1-(v/c)^2}$, and its Coulomb-energy in the electric field of the proton. On the other hand we estimate $\alpha$ by

$$\alpha \approx \frac{1}{m_e}(m_e r(r\underbrace{\frac{d\phi}{dt}}_{v_\phi \approx v})\cdot\frac{1}{\sqrt{1-(v/c)^2}}), \qquad (34)$$

where we used equation (2) and the special relativistic time dilation relation $d\tau = dt\sqrt{1-(v/c)^2}$. For $r$ we take Bohr's radii $r_n = \varepsilon_0 h^2/(\pi e^2 m_e)\cdot n^2$, where $n$ is the main quantum number of the energy levels in the hydrogen atom and $h$ is Planck's constant. For $v$ we take the corresponding estimates of the electron velocities in Bohr's model of the atom $v_n = e^2/(2\varepsilon_0 hn)$. Note that despite the fact that $r_n$ depends on $h^2$ in our formula, $h^2$ can be expressed by the energy of the corresponding level in the atom. Thus, $r_n$ does not really depend on $h^2$. Yet, $h$ determines the rotational momentum of the electron via the quantization condition $2\pi r_n m_e v_n = nh$, and this is the way how $h$ has been introduced to micro physics by Niels Bohr historically. This basically means that we have $\alpha = n\hbar/m_e$, i.e. $\alpha$ is a multiple of $\hbar = h/(2\pi)$, normalized to the electron mass.

Next, we calculate all the constants needed for the ground state of the hydrogen atom, i.e. for $n = 1$. In SI-units we get the following numerical values:
$\kappa^2 = 2.07593066\times10^{-43}$, $k = 1.43998357\times10^{-9}$,
$m = 1.24167118\times10^{-54}$,
$q = -5.86680189\times10^{2}$,
$r = r_{\text{Bohr}} = 5.29176489\times10^{-11}$, $v = 2.18770919\times10^{6}$,
$\alpha = 1.15771509\times10^{-4}$, $\beta = 2.99784518\times10^{8}$,
$a_1 = 2.15227578\times10^{-61}$, $\bar{a} = -1.15382164\times10^{7}$,
$\bar{b} = 4.64599732\times10^{60}$,
$\bar{c} = -1.75588816\times10^{71}$ and $\bar{d} = 1.65903443\times10^{81}$.

In order to calculate the four roots of our polynomial of degree four $V_{pseudo}(u,\beta,\alpha)$, we use the root-solver "roots(*)" of REDUCE. The result for the ground state is

$$u_0 = 1.889628064590297145\times10^{10}$$
$$u_1 = 1.889728703523198162\times10^{10}$$

$$\mu = -1.889101473236373256\times10^{10}$$

$$\nu = 2.155457567587315225\times10^{30},$$

i.e. there are two complex conjugate roots and the motion is only between $u_0$ and $u_1$. For the aphelion we get $r_0 = 1/u_0 =$ $5.29204672\times10^{-11}$ m. For the perihelion we get $r_1 = 1/u_1 = 5.29176489\times10^{-11}$ m, whitch is identical with Bohr's radius. The coordinate of the aphelion is only slightly larger than Bohr's radius. Thus, we get almost a circular trajectory of the electron around the proton. If we modify the parameters $\alpha$ and $\beta$ slightly, we get values $r_0$ and $r_1$ that do not coincide so closely. Hence, there is a radial motion between aphelion and perihelion in those cases, which is the wobble motion that we have already discussed. Such a trajectory is not suiting for a landing destination of electrons in the hydrogen atom that can facilitate coherent emission from the electrons. On such a trajectory, energy can only be dissipated and these processes do not contribute to the stability of the atom.

The reader might wonder that we argue with classical trajectories within the atom here. However, in the light of the discussion in Schmidt, (2016), the propagating electron moves through a "Dirac ocean" of proton anti-proton pairs. It can regularly plunge into one of the protons it encounters. The plunging has the result that the re-emerging electron is displaced temporally and spatially. Thus, the electron exhibits a fluctuating behaviour all the time. However, this statistical broadening does not hinder that on average, the guiding center of the fluctuating electron still follows the classical trajectory. In this respect it makes sense to investigate classical trajectories in the atom.

We can extend this investigation to higher energy levels. E.g., for $n = 2$ we get

$$u_0 = 4.724258862614892384\times10^{9}$$
$$u_1 = 4.724321758808099473\times10^{9}$$

$$\mu = -4.718521202507751960\times10^{9}$$
$$\nu = 2.155500610165894935\times10^{30}$$
$$r_0 = 2.11673414\times10^{-10}\,\text{m}$$
$$r_1 = 2.11670596\times10^{-10}\,\text{m}$$

Again, $r_1$ is four times Bohr's radius, and $r_0$ is so close to $r_1$ that we get in fact a circular trajectory. If we perturb the parameters $\alpha$ and $\beta$ again, we get a much less prominent fit of $r_0$ and $r_1$. Thus, also in these csaes a wobble motion is on top of the circular motion, which does not admit for coherent emission of electrons that land on such a trajectory.

We have also investigated the case $n = 3$. Here, we get

$$u_0 = 2.099686135645422301\times10^{9}$$
$$u_1 = 2.099698559470529518\times10^{9}$$
$$\mu = -2.093923239354231939\times10^{9}$$
$$\nu = 2.155508581296781307\times10^{30}$$
$$r_0 = 4.76261658\times10^{-10}\,\text{m}$$

$$r_1 = 4.7625884\times10^{-10}\,\mathrm{m}$$

Again, we obtain an almost ideal circular orbit. $r_0$ and $r_1$ are nine times Bohr's radius. Perturbations of $\alpha$ or $\beta$ lead to a wobble motion on top of this circular orbit.

We now investigate the numerical values we found for $\alpha_n$ and $n = 1,2,3$. We obtain

$$2\pi m_e(\alpha_{n=2} - \alpha_{n=1}) \approx 6.6262\times10^{-34}$$
$$2\pi m_e(\alpha_{n=3} - \alpha_{n=2}) \approx 6.6262\times10^{-34}$$

The numerical value on the right side is the observed value for the Planck constant . Thus, we have confirmed that in the general relativistic atom circular orbits exist that correspond to Bohr's trajectories. We have further confirmed that for the first two steps in these circular orbits with ascending radius, the rotational momentum between these circular orbits is incremental, whereby the increment between neighbouring circular orbits is a constant. This constant is identical with the measured Planck constant.

With this, we have found a numerical method to derive the Planck constant. This goes as follows: We perform a numerical parameter study, where our parameters are the and rotational momentum and energy parameters. We start with pairs of and , and look if the two real roots of the pseudo potential are close to each other. If they are not close, or must be modified in order to obtain a better match. In this way we converge on one of Bohr's orbit. We can converge on all of Bohr's orbits in this way. The result is a rotational momentum ladder spectrum, which increases by a constant when we look at ascending neighbouring circular orbits, or in other words the rotational momentum ladder spectrum is equidistant. This constant can be determined by solely carrying out numerical experiments. However, as we have shown, this constant is identical with the Planck constant that we know from the experiments.

We can expand on this thought further. We can imagine that one of the other natural constants that enter the calculation has a slightly different value (it is the electron mass and charge, the proton mass and charge, Newton's gravitational constant, the dielectric constant, and the light velocity). Then, the solution of the atom with circular orbits would not yield an equidistant rotational momentum ladder spectrum. Hence, coherent emission from such a system was not possible, and such an atom must necessarily dissipate. Hence, the values of the natural constants as they are observed in the experiments are the only values that enable a stable atom and are therefore unique.

## CONCLUSIONS

We have investigated the geodetic motion with rotational momentum in the Kaluza Klein theory with minimally coupled gravitational and electromagnetic fields. We have shown that the orbits follow generalized Kepler ellipses, where these orbits undergo a rotation of the perihelion and the ellipses are disturbed by an additional wobbel motion in the radial direction. This disturbance is caused by a rotational potential that becomes gravitationally active. When the masses of the particles envolved is increased, this rotational potential can amplify to a potential barrier that devides the orbit into a confined inner orbit and an outer orbit. When we apply these findings to micro physics, the generalized Kepler ellipses describe the electro magnetic interaction force, whereas the cases with increased masses organically belong to the electro strong and electro weak interaction forces. In the case of the electro magnetic interaction force we can look for ideal circular orbits. We find that discrete such orbits exist and that they coincide with Bohr's circular orbits in the atom. Quite naturally, from these findings a numerical procedure can be defined in order to determine the Planck constant numerically, which is the constant increment in rotational momentum between adjacent circular orbits. In order that such a constant increment exists, which guarantees the stability of the atom, none of the other natural constants can have a deviating value to the value taken from the measurements.

## REFERENCES

[1] Misner, Charles, Thorne, Kip S., & Wheeler, John Archibald (1973), Gravitation, San Francisco, W. H. Freeman, ISBN 0-7167-0344-0.

[2] Schmidt, J.M. (2016), Space travel of charged test particles hitting the singularity of a charged central gravitational center, International Journal of Technical Research and Applications (IJTRA), vol. 4, issue 6, Nov-Dec 2016