MAGNETIC FIELD EFFECT ON TWO LAYEREDMODEL OF BLOOD FLOW

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Abstract— **In this paper a two layered magneto hydrodynamic flow through parallel plates under the transverse magnetic fluid has been investigated.Expressions for velocity profile in the core region and peripheral plasma layer regions , flow rates and effective viscosity has been obtained.Variation of flow rates and effective viscosity with PPL thickness for different values of Hartmann number is shown with the helpof tables and graphs.**

Index terms-**Hartmann number , Effective viscosity,Magnetic field,Variation of flow rates**

I. INTRODUCTION

A device used to remove metabolic wastes and other unwanted chemicals from human blood are known as dialyser or artificial kidney. There are many types of dialyser, for example flat plate, tubular and helical. The fluid mechanical aspects of flat plate type dialyser have been dealt with in the present chapter. Gupta and Sheshadri [4] has represented the flow of RBC suspension through narrow tube Charm and Kurland [2], Bugliarello and Sevilla [1], Kiani and Hudetz [5] have propagated two layered flow mode] of blood with marginal plasma layer near the wall. Chaturani and Bhartiya [3] have studied the two layered magneto hydrodynamic flow through parallel plates with application. Recently Mishra et al. [6] have studied "A rheological fluid model for magneto hydrodynamics parallel plate's hemodialyser". Since blood containing RBC has contain hemoglobin which is a magnetic material, therefore it may be useful to impose a magnetic field in the flow field which pushes red cells away from the dialyser membrane, which increase the dialysis area and decrease the dialysis time.

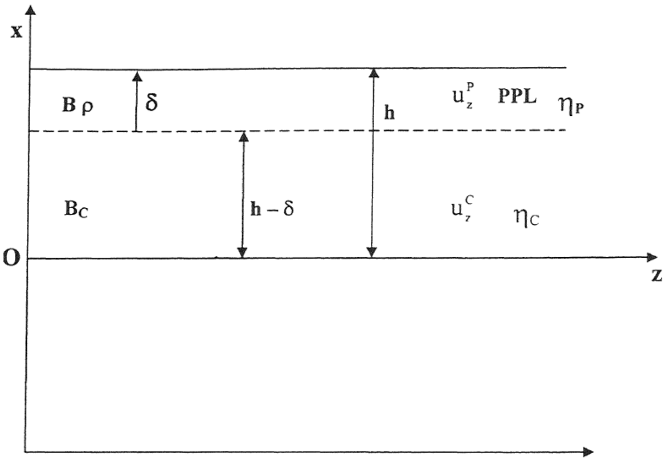
Here in this chapter a two layered magneto hydro- dynamic flow through parallel plates under the transverse magnetic fluid has been investigated. Expressions for velocity profiles in the core region and peripheral plasma layer regions, flow rate and effective viscosity have been obtained. Variation of flow rate and effective viscosity with PPL thickness for different values of Hartmann number is shown with the help of Tables and Graphs.

II. ANALYSIS:

Consider a steady incompressible laminar viscous flow of blood between two parallel plates. Further it is assumed that the length and width of the plates are very large in comparison to the distance between them. So the problem may be approximated as one dimensional flow between two infinite parallel plates at distances 2h apart. Also blood is assumed to be Newtonian fluid.

The flow is taken along z-direction and constant uniform magnetic field is applied in x direction. So that z-axis is parallel to the flow velocity Uz.

We shall consider steady flow where velocity $U_z = u_z(x)$ and magnetic induction vector B = B(x)



Figure

Governing equations of motion for steady incompressible flow are

Governing equations of motion for steady incompressible flow are

$$(u\cdot\nabla)u - \text{grad}\left(P + \frac{\mu H^2}{8\pi}\right) + \frac{\mu}{4\pi}(H\cdot\nabla) + \eta\nabla^2 u \quad \text{Momentum equation} \tag{1.1}$$

$$\text{div } u = 0 \quad \text{Continuity equation} \tag{1.2}$$

$$\text{div } H = 0 \quad \text{Maxwell equation} \tag{1.3}$$

$$(H\cdot\text{grad})u - (u\cdot\text{grad})H - \text{curl}(\eta_m \text{ curl } H) = 0 \tag{1.4}$$

Magnetic field equation

where δ is PPL thickness, $u_z^{(P)}$ and $u_z^{(C)}$ are plasma and core velocities respectively, μ is magnetic permeability, η – fluid viscosity, η_m – magnetic viscosity, $H = B/\mu$ – magnetic field vector and other symbol have their usual meaning then

Equation (1.1) along x, y and z direction reduces to

$$0 = -\frac{\partial}{\partial x}\left(P + \frac{\mu H^2}{8\pi}\right) + \frac{\mu}{4\pi} H_x \frac{\partial H_x}{\partial x} \quad (1.5)$$

$$0 = \frac{\mu}{4\pi} H_x \frac{\partial H_y}{\partial x} \quad (1.6)$$

$$0 = -\frac{\partial P}{\partial z} + \frac{\mu}{4\pi} H_x \frac{\partial H_z}{\partial x} + \eta \frac{\partial^2 u_z}{\partial x^2} \quad (1.7)$$

Equation (1.3) assumes the form

$$\frac{\partial H_x}{\partial x} = 0 \quad (1.8)$$

Magnetic field equation (1.4) gives

$$0 = \eta_m \frac{\partial^2 H_x}{\partial x^2} \quad (1.9)$$

$$0 = \eta_m \frac{\partial^2 H_y}{\partial x^2} \quad (1.10)$$

$$0 = H_x \frac{\partial u_z}{\partial x} + \eta_m \frac{\partial^2 H_z}{\partial x^2} \quad (1.11)$$

From equation (4.8) we get

$$H_x = \text{constant} = H_0 \quad (1.12)$$

The system of partial differential equations (1.7) and (1.11) reduces to

$$\eta \frac{d^2 u_z}{dx^2} + \frac{\mu H_0}{4\pi} \frac{dH_z}{dx} + K_1 = 0 \quad (1.13)$$

$$H_0 \frac{du_z}{dx} + \eta_m \frac{d^2 H_z}{dx^2} = 0 \quad (1.14)$$

where $K_1 = -\frac{dP}{dz}$

If we put $v, w = u_z \pm \frac{\alpha \eta_m}{\eta} H_z$ (1.15)

where $\alpha = \left(\frac{\mu \eta}{4\pi \eta_m}\right)^{1/2}$

then we find
$$\eta \frac{d^2 v}{dx^2} + \alpha H_0 \frac{dv}{dx} + K_1 = 0 \quad (1.16)$$

and
$$\eta \frac{d^2 w}{dx^2} - \alpha H_0 \frac{dw}{dx} + K_1 = 0 \quad (1.17)$$

with the help of boundary conditions

$$\left.\begin{array}{ll} u_z(\pm h) = 0, & B_z(\pm h) = 0 \\ u_z^{(P)} = u_z^{(C)}, & B_z(x) = 0 \quad \text{at } x = \pm(h-\delta) \end{array}\right\} \quad (1.18)$$

we solve equations (1.16) and ((1.15) by using transformations (1.15) and get

$$u_z^{(P)} = \frac{K_1 d^2}{2\eta_p}\left(1 - \frac{x^2}{h^2}\right) \quad (h-\delta) \le x \le h, \quad -h \le x \le -(h-\delta) \quad (1.19)$$

$$u_z^C = \frac{K_1 h^2}{2\eta_p}\left[\left(\frac{\delta}{h}\right)\left(2 - \frac{\delta}{h}\right)\right]$$

$$+ \frac{K_1 h^2}{\eta_C M}\left(1 - \frac{\delta}{h}\right)\left[\frac{\cosh M\left(1 - \frac{\delta}{h}\right) - \cosh M\left(\frac{x}{h}\right)}{\sinh M\left(1 - \frac{\delta}{h}\right)}\right]$$

$$, \quad -(h-\delta) \le x \le (h-\delta) \quad (1.20)$$

The core velocity for two limiting cases be obtained as

Case – 1 : M ⟶ 0

$$u_z^{(C)} = \frac{K_1 h^2}{2\eta_p}\left[\left(\frac{\delta}{h}\right)\left(2 - \frac{\delta}{h}\right)\right] + \frac{K_1 h^2}{2\eta_C}\left[\left(1 - \frac{\delta}{h}\right)^2 \frac{x^2}{h^2}\right] \quad (1.21)$$

Case – 2 M >> 1

$$u_z^{(C)} = \frac{K_1 h^2}{2\eta_p}\left[\left(\frac{\delta}{h}\right)\left(2 - \frac{\delta}{h}\right)\right] + \frac{K_1 h^2 \left(1 - \frac{\delta}{h}\right)}{\eta_C M}\left[1 - e^{-m\left[\left(1-\frac{\delta}{h}\right) - \frac{x}{h}\right]}\right]$$

Now we calculate flow rate and effective viscosity.

(i) Flow rate : The flow rate Q is defined as

$$Q = \int_{-h}^{h} u\,dx$$

$$= \int_{-h}^{-(h-\delta)} u_z^{(P)}\,dx + \int_{-(h-\delta)}^{0} u_z^{(C)}\,dx + \int_{0}^{(h-\delta)} u_z^{(C)}\,dx + \int_{h-\delta}^{h} u_z^{(P)}\,dx$$

which on substituting $u_z^{(C)}$ and $u_z^{(P)}$ from equations (1.19) and (1.20)) gives

$$Q = -\frac{K_1 h^3}{\eta_p}\left(\frac{\delta}{h}\right)^2\left[\frac{1}{3}\left(\frac{\delta}{h}\right) - 1\right]$$

$$+ \frac{K_1 h^3}{\eta_p}\left(\frac{\delta}{h}\right)\left(1-\frac{\delta}{h}\right)\left(2-\frac{\delta}{h}\right)$$

$$+ \frac{2K_1 h^3}{\eta_c M^2}\left(1-\frac{\delta}{h}\right)\left[M\left(1-\frac{\delta}{h}\right)\coth M\left(1-\frac{\delta}{h}\right) - 1\right] \qquad (1.22)$$

In the case $M \to 0$

$$Q = -\frac{K_1 h^3}{\eta_p}\left(\frac{\delta}{h}\right)^2\left[\frac{1}{3}\left(\frac{\delta}{h}\right) - 1\right]$$

$$+ \frac{K_1 h^3}{\eta_p}\left(\frac{\delta}{h}\right)\left(1-\frac{\delta}{h}\right)\left(2-\frac{\delta}{h}\right) - \frac{2K_1 h^3}{3\eta_c}\left(1-\frac{\delta}{h}\right)^3 \qquad (1.23)$$

For one layered MHD model flow rate Q is given by

$$Q = \frac{2K_1 h^3}{\eta_c M^2}\left[M \coth M - 1\right] \qquad (1.24)$$

Effective Viscosity :

The expressions for the effective viscosity for a two layered MHD model is given by

$$\mu_{eff} = \frac{1}{\frac{1}{\eta_p}\left[\left(\frac{\delta}{h}\right)^2\left\{1 - \frac{1}{3}\left(\frac{\delta}{h}\right)\right\} + \frac{\delta}{h}\left(1-\frac{\delta}{h}\right)\left(2-\frac{\delta}{h}\right)\right] + \frac{2}{\eta_c M^2}\left(1-\frac{\delta}{h}\right)\left[M\left(1-\frac{\delta}{h}\right)\coth M\left(1-\frac{\delta}{h}\right) - 1\right]} \qquad (1.25)$$

As $M \to 0$ i.e. for non-magnetic two layered model the effective viscosity can be obtained from equation (1.25) and is given by

$$\mu_{eff} = \frac{1}{\frac{1}{\eta_p}\left[\left(\frac{\delta}{h}\right)^2\left\{1 - \frac{1}{3}\left(\frac{\delta}{h}\right)\right\} + \frac{\delta}{h}\left(1-\frac{\delta}{h}\right)\left(2-\frac{\delta}{h}\right)\right] - \frac{2}{3\eta_c}\left(1-\frac{\delta}{h}\right)^3} \qquad (1.26)$$

For one layered MHD model apparent viscosity is given by

$$\mu_{eff} = \frac{1}{\frac{2}{\eta_c M^2}\left[M \coth M - 1\right]} \qquad (1.27)$$

Table- 1.1

Variation of flow rate with PPL thickness for constant value of Hartmann number.

δ/h	Q		
	M = 1	M = 2	M = 3
0.01	0.0009	0.0008	0.0006
0.02	0.0010	0.0008	0.0007
0.03	0.0011	0.0009	0.0008
0.04	0.0012	0.0010	0.0009
0.06	0.0012	0.0011	0.0010
0.08	0.0014	0.0013	0.0012

Table-1.2

Variation of flow rate with PPL thickness in case of non-magnetic field (i.e. $M \to 0$)

δ/h	Q
0.01	– 0.00071
0.02	– 0.00058
0.03	– 0.00045
0.04	– 0.00033
0.06	– 0.00010
0.08	0.00011
0.10	0.00016

Table- 1.3

Variation of flow rate with Hartmann number M for one-layered model (i. e. δ/h = 0)

M	Q
1	0.0033
2	0.0027
3	0.0023
4	0.0019
5	0.0015

Table-1.4

Variation of effective viscosity with PPL thickness for different values of Hartmann number.

δ/h	η_{eff}		
	M = 1	M = 2	M = 3
0.01	0.0072	0.0082	0.0096
0.02	0.0066	0.0074	0.0083
0.03	0.0060	0.0065	0.0072
0.04	0.0056	0.0060	0.0066
0.06	0.0050	0.0054	0.0058
0.08	0.0045	0.0048	0.0051

Table-1.5

Variation of effective viscosity with PPL thickness in case of non- magnetic field (i.e. M 0)

δ/h	η_{eff}
0.01	– 0.008
0.02	– 0.010
0.03	– 0.013
0.04	– 0.018
0.05	– 0.028
0.06	– 0.060

Table- 1.6

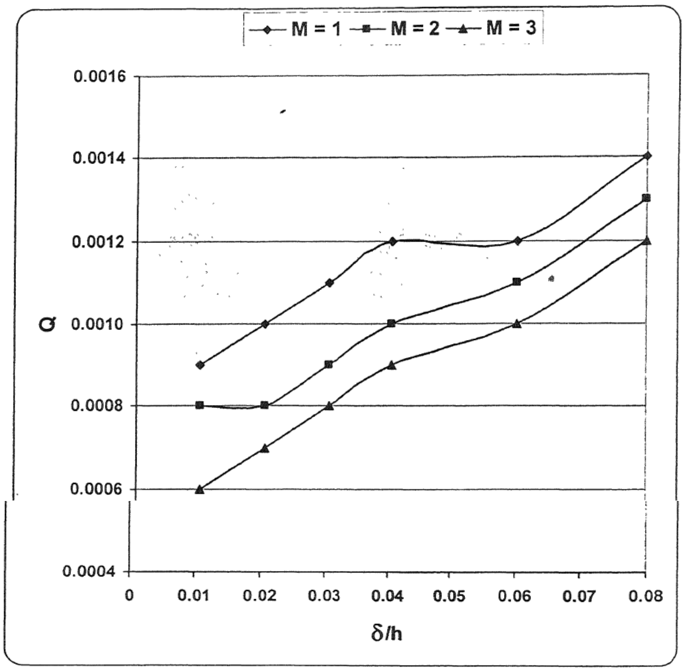
Variation of effective viscosity with Hartmann number M for one- layered model (i.e. 8/h = 0)

M	η_{eff}
1	0.0019
2	0.0022
3	0.0027
4	0.0032
5	0.0037

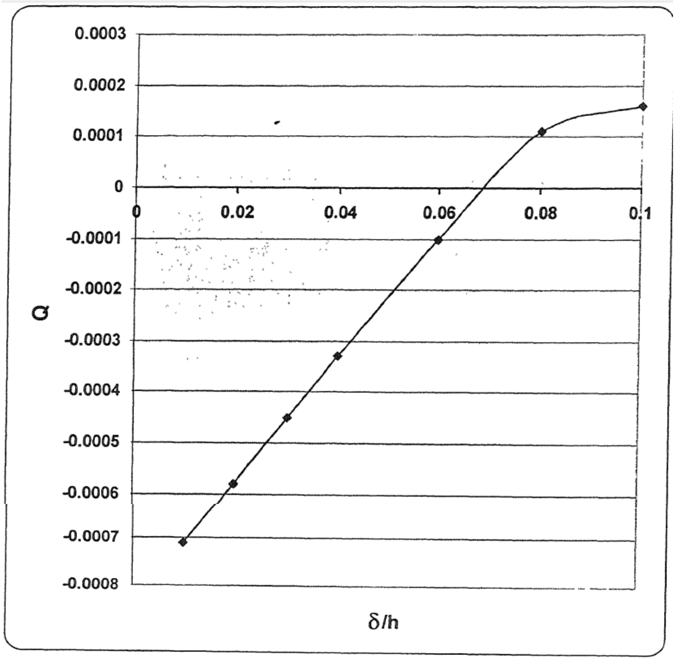
Table- 1.7

Variation of viscosity profile for one layered model (8/h = 0.0)

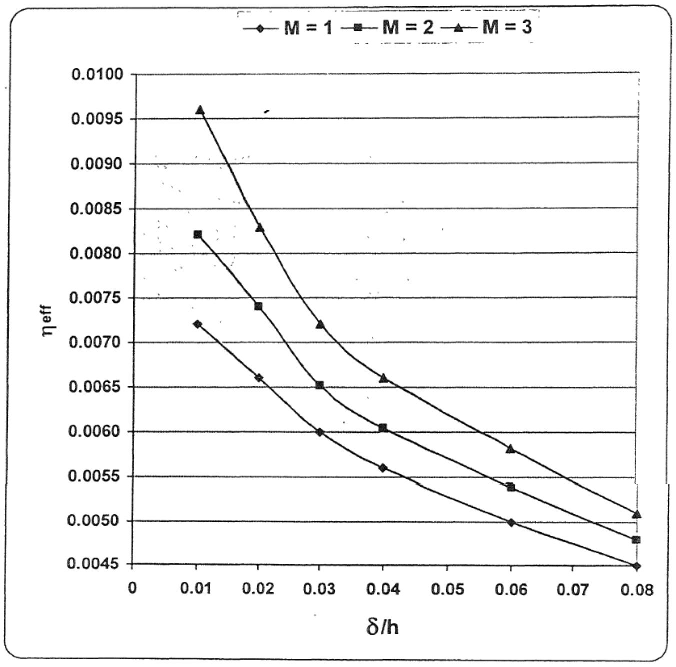
x/h	$u_z^{(C)}$			
	M = 1	M = 2	M = 3	M = 10
0.0	2.4064	3.9665	4.7142	5.2078
0.2	2.3178	3.8502	4.6178	5.2065
0.4	2.0475	3.4825	4.2928	5.1953
0.6	1.5865	2.8024	3.6186	5.1129
0.8	0.9129	1.7012	2.3451	4.5034
1.0	0.0000	0.0000	0.0000	0.0000



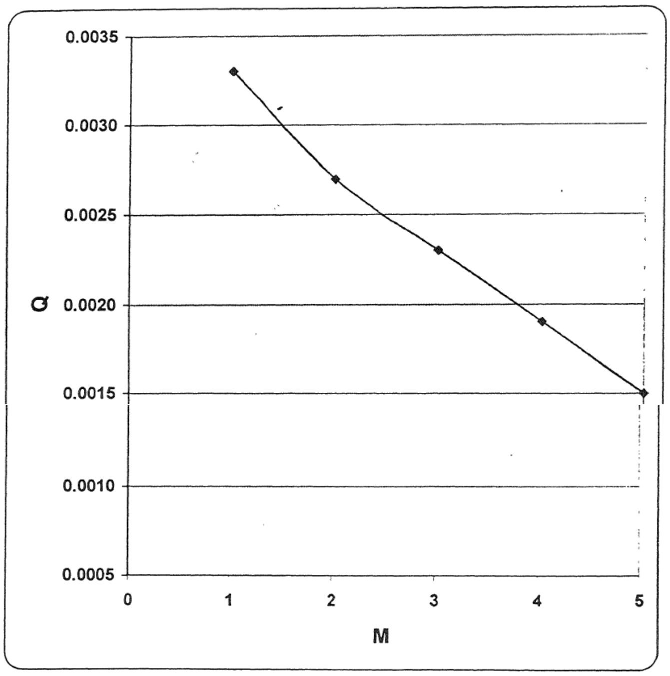
Graph – 1.1: Variation of flow rate with PPL thickness.



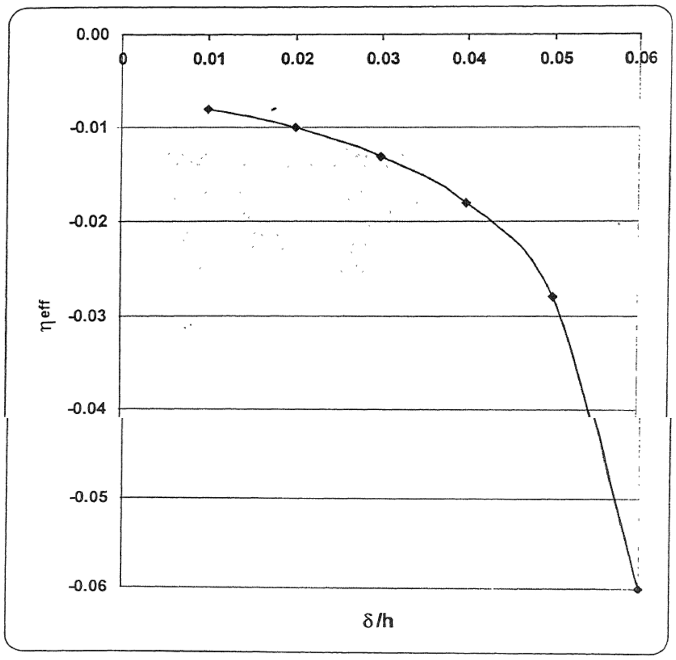
Graph – 1.2: Variation of flow rate with PPL thickness ($m \rightarrow 0$).



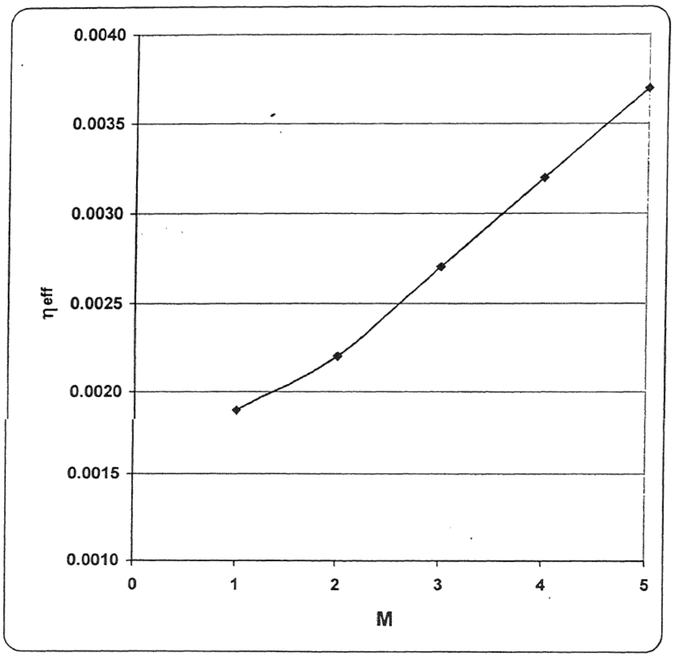
Graph – 1.4: Variation of effective viscosity with PPL thickness.



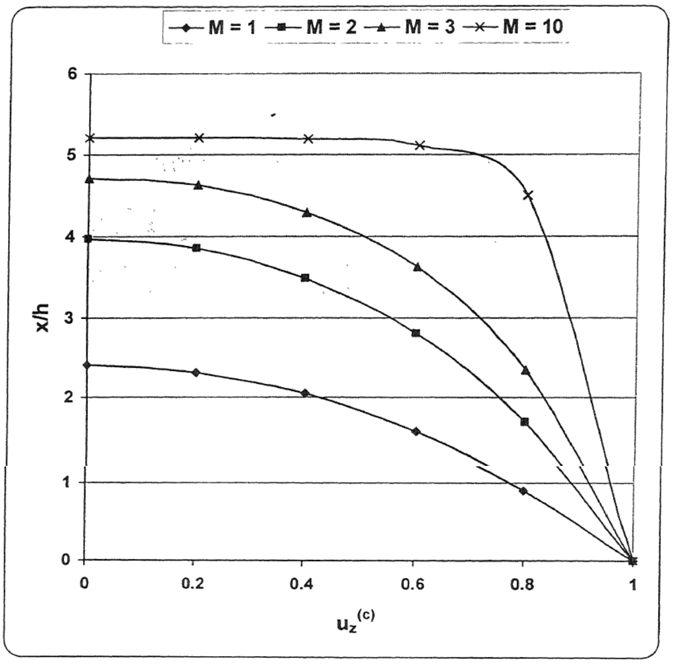
Graph – 1.3: Variation of flow rate with Hartman number.



Graph – 1.5: Variation of effective viscosity with PPL thickness ($M \rightarrow 0$).



Graph – 1.6: Variation of effective viscosity with Hartmann number.



Graph – 1.7: Variation of velocity for different values of PPL thickness..

i. It is shown from Table - 1.25 that flow rate is increases with the increase in PPL thickness for constant Hartmann number M. Also the flow rate decreases with increase in the Hartmann number at constant PPL thickness.

ii. In case of non-magnetic field flow rate also increases with increase in PPL thickn 1.25 is shown from Table – 1.2

iii. In case of one-layered model i.e. $\frac{\delta}{h} = 0$ flow rate decreases with increase in Hartman number that is shown from Table – 1.3

iv. From Table – 1.4 we see that effective viscosity increases with increases Hartmann number for constant PPL thickness. But effective viscosity decreases with increase in PPL thickness at constant Hartmann number.

v. From Table – 1.5 we see that effective viscosity decreases with increase in PPL thickness, when there is no magnetic field.

vi. For one layered model (i.e. $\frac{\delta}{h} = 0$) effective viscosity increases with increase in Hartmann number M, it is shown by Table – 1.6

vii. The velocity profiles of a one-layered MHD flow can be obtained as a special case of the present model by substituting $\delta = 0$ in equation (1.20). The effect of Hartmann number on this profile is shown in Graph – 1.6 It is observed that decrease in Hartmann number M leads to flattening of the profiles. For very low values of M (= 0) the flow is almost stagnant.

III. CONCLUSION

.Since blood containing RBC contain hemoglobin which is magnetic material , therefore it may useful to impose a magnetic field in the flow field which pushes redcells away from the dialyser membrane ,which increase the dialysis area and decrease the dialysis time.

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