A STUDY ON QUEUING MODEL TO REDUCE THE WAITING TIME IN FUEL STATIONS

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Abstract— **In many fuel stations there had always been long queues to collect the fuel as a result there may be a scarcity of the product in recent times. This type of situation makes the buyers to wait for more than hours. Especially the scarcity is higher in the festive periods. The service station in cities sells products to the available customers in a random order, which results to long queue in fuel stations. In this paper a single channel multiple – server model is considered to reduce the waiting time of the customers.**

Keywords— Queuing Model, Kendall's Notation, Poisson Probability distribution, Average Waiting time.

I. INTRODUCTION

In queuing theory we use the Kendall's notation (a/b/c/d/e/f) to describe the queue model. The arrival, service and departure of the customers are independent to one another. This process is always continues. The arrivals per unit length of time are calculated using Poisson probability distribution. The queue discipline is FCFS (First Come First Serve) with infinite queue system.

This ensures that the waiting time of the customers waiting time of the customers is greatly reduced **[2]**. The search to stop the frequently arising problem of delays experienced in the queuing system in cities. As a result a long queue is normally exists in fuel stations on daily bases.

Scarcity of fuel products from the supply will results in

(i)   Creating confusions among the customers.
(ii)   Customers have to wait for a long time without being served.
(iii) Returning of the customers.

Mainly, Queuing system is associated to arrivals and services. Not only in fuel stations but also in road signals there are queues. It represents a sequence of public, vehicles, etc., waiting for their turn to service.

Queuing system has two basic forms namely called Structured and Unstructured queue **[1]**. Moreover queuing theory is the study on these situations from a mathematical point of view. In recent times queue models are used to reduce the average waiting time in Traffic and delays in communication networks, service stations, etc.,

One method to analyze these system components is to approximate them by tractable objects. Queuing systems are part of our daily activities its application is widely pronounced in banking sectors, filling stations, hospitals, traffic light junctions etc., and it may not be avoided. However the good queuing management technique will control the Traffic by the reduction of waiting time.

Queue theory has all the required tools to avoid the unwanted delays. This work develops suitable queuing models for the fuel stations. The data which we are using here is collected in between the time interval 7 a.m. to 12 p.m. for three days.

II. DESCRIPTION OF THE MODEL AND THE DATA

Three days data was collected from fuel station. The arrival time of customers was calculated by the help of stop – watch. Analysis of this type is helpful in the calculation of inter –

arrival time and service time. A simple activity of the queuing system is given below:
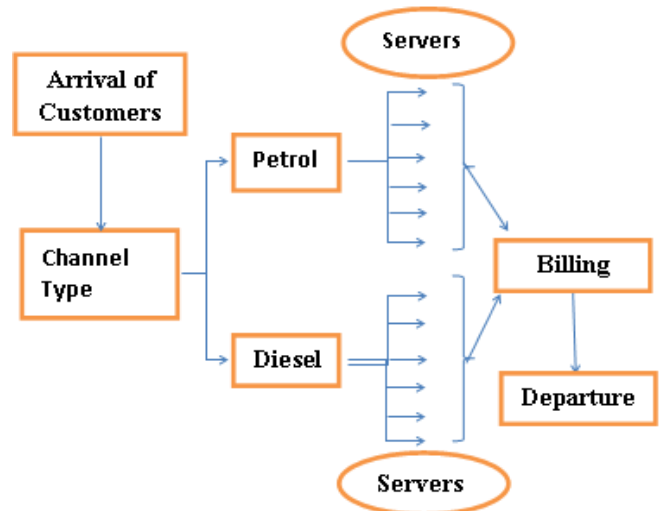
System analysis describes the activity and to find the problem area to provide immediate solution by reducing the waiting time. Let S_1, S_2, S_3 be the observed Inter – arrival time for day 1, 2, 3 respectively.

Table 1: Results of Inter - Arrival time / frequencies for 3 days							
Inter arrival time	Service Time (T)	S_1	S_1t	S_2	S_2t	S_3	S_3t
1 – 10	2.5	50	125	60	150	45	112.5
11 – 20	12.5	42	525	53	662.5	32	400
21 – 30	22.5	33	742.5	45	1012.5	28	630
31 – 40	32.5	21	682.5	30	975	19	617.5
41 – 50	42.5	15	637.5	21	892.5	10	425
51 – 60	52.5	15	787.5	16	840	15	787.5
61 – 70	62.5	12	750	8	500	2	125
71 – 80	72.5	5	362.5	4	290	3	217.5
81 – 90	82.5	8	660	9	742.5	8	660
91 – 100	92.5	2	185	7	647.5	1	92.5
101 – 110	102.5	1	102.5	3	307.5	5	512.5
111 – 120	112.5	16	1800	10	1125	7	787.5
121 – 130	122.5	3	367.5	2	245	1	122.5
131 – 140	132.5	1	132.5	5	662.5	8	1060
141 – 150	142.5	5	712.5	2	285	1	142.5
151 – 160	152.5	1	152.5	2	305	9	1372.5
161 – 170	162.5	1	162.5	1	162.5	5	812.5
171 – 180	172.5	7	1207.5	8	1380	2	345
Total		**238**	**10095**	**286**	**11185**	**201**	**9222.5**

Calculating the Inter – arrival time

For day 1:

Mean Inter – arrival time is

$$t_1 = \frac{\sum S_1 t}{\sum S_1} = \frac{10095}{238} = 42.4 \approx 42$$

$$t_1 = 42 \sec / bike$$

Arrival Rate is

$$\lambda_1 = \frac{1}{t_1} = \frac{1}{42} = 0.0238 \ bike / \sec$$

$$= 2.38 \ bike / \min \approx 2$$

$$\lambda_1 = 2 \ bike / \min$$

For day 2:

Mean Inter – arrival time is

$$t_2 = \frac{\sum S_2 t}{\sum S_2} = \frac{11185}{286} = 39.1083 \approx 39$$

$$t_2 = 39 \sec / bike$$

Arrival Rate is

$$\lambda_2 = \frac{1}{t_2} = \frac{1}{39} = 0.0256 \ bike / \sec$$

$$= 2.56 \ bike / \min \approx 2$$

$$\lambda_2 = 2 \ bike / \min$$

For day 3:

Mean Inter – arrival time is

$$t_3 = \frac{\sum S_3 t}{\sum S_3} = \frac{9222.5}{201} = 45.8835 \approx 46$$

$$t_3 = 46 \sec / bike$$

Arrival Rate is

$$\lambda_3 = \frac{1}{t_3} = \frac{1}{46} = 0.0217 \ bike / \sec$$

$$= 2.17 \ bike / \min \approx 2$$

$$\lambda_3 = 2 \ bike / \min$$

The Average Inter – arrival time is given by

$$T = \frac{t_1 + t_2 + t_3}{3} = \frac{42 + 39 + 46}{3} = 42.333$$

$$T = 42 \ \sec / bike$$

Calculation for Average Arrival Rate

$$\lambda = \frac{1}{T} = \frac{1}{42} = 0.0238 = 0.02 \ bike / \sec \approx 2$$

$$\lambda = 2 \ bike / \min$$

Let F_n , n= 1,2,3,4,5,6 be the service time for servers 1 to 6.

Table 2:
Result of computation for t and F_n for servers 1 to 6

Service time interval (Mints)	F₁	F₂	F₃	F₄	F₅	F₆
0-4	51	49	60	58	54	65
5-9	20	24	18	15	22	21
Total	**71**	**73**	**78**	**73**	**76**	**86**
	F₁t	F₂t	F₃t	F₄t	F₅t	F₆t
0-4	102	98	120	116	108	130
5-9	140	168	126	105	154	147
Total	**242**	**266**	**246**	**221**	**262**	**277**

Let t_m be the mean service time

$$t_m = \frac{\sum S_m t}{\sum S_m}$$

The service rate is given by $\mu_n = \frac{1}{t_m}$.

Table 3: Result of t_m and μ_s
for the 6 servers

No. of Servers	t_m (Mints / car)	μ_s (cars /Mints)
1	4.5	0.23
2	4.2	0.24
3	4.76	0.21
4	4.23	0.24
5	4.36	0.23
6	4.77	0.2

Average service time T_m is given by

$$T_m = \frac{\sum t_m}{m}$$

$$= \frac{t_1 + t_2 + t_3 + t_4 + t_5 + t_6}{6} = \frac{4.5 + 4.2 + 4.76 + 4.23 + 4.36 + 4.77}{6}$$

$$= \frac{26.82}{6} = 4.47$$

$$\approx 4 \ \min / bike$$

The average service rate μ_s is given by

$$\mu_S = \frac{1}{T_m} = \frac{1}{4} = 0.25 \ car / \min$$

III. M/M/S QUEUE MODEL

The first known values in a calculation of performance measures is,

  (i) Traffic intensity (ρ)

  (ii) Probability of the system should be idle (P_0)

The traffic intensity $\rho = \frac{\lambda}{s\mu}$

λ = Arrival rate per minute

s = Number of servers
μ = Service rate per minute

$$\rho = \frac{2.38}{(6)(0.25)} = 1.5866$$

$$\approx 1.6$$

The probability that the system should be idle

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^n \frac{s\lambda}{s\mu - \lambda} \right]^{-1}$$

$$= \left[\sum_{n=0}^{6-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{6!}\left(\frac{\lambda}{\mu}\right)^6 \frac{s\lambda}{s\mu - \lambda} \right]^{-1}$$

$$= \left[\sum_{n=0}^{5} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{6!}\left(\frac{2.38}{0.25}\right)^6 \frac{6(0.25)}{6(0.25) - 2.38} \right]^{-1}$$

$$P_0 = 0.0005676$$

Hence the percent of idleness in the system will be 5.6%.

To find the expected values
Number of customers waiting in the queue L_q is given by

$$L_q = \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu - \lambda)^2} \right] P_0$$

$$= \left[\frac{1}{5!}\left(\frac{2.38}{0.25}\right)^5 \frac{(2.38)(0.25)}{(-0.88)^2} \right] P_0$$

$$= \left[\frac{1}{5!}(78196.037)(0.043) \right] \frac{5.6}{100}$$

$$= 28.02$$

$$L_q \approx 28$$

Number of customers waiting in the system L_s is given by

$$L_s = L_q + \left(\frac{\lambda}{\mu}\right) = 28.02 + 9.52 = 37.5$$

$$L_s \approx 37$$

Waiting time of customers in the queue W_q is given by

$$W_q = \frac{L_q}{\lambda} = \frac{28}{2.38} = 11.76$$

$$W_q \approx 12$$

Waiting time of customers in the system W_s is given by

$$W_s = W_q + \left(\frac{1}{\mu}\right) = 12 + \frac{1}{0.25}$$

$$W_s = 16$$

IV. CONCLUSION
In this we simulate a single line multi – server queuing system. One single line multi – server queuing system will handle infinite queue length.
Queuing system is designed to provide
 (i) Fast
 (ii) Efficient
 (iii) Comfortable
 (iv) Cost effective and
 (v) Safe services.
The system eliminates
 (i) Time wastage
 (ii) Space constraints
 (iii) Risk
To this end, by the implementation of queuing system a different dimension in industries will occur.

References
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