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UNDERSTANDING OF STUDENTS ON LINEAR EQUATIONS THROUGH ANALYSIS OF SELF-MADE QUESTIONS

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Abstract—Understanding is the connection of concepts that forms a general idea. Educational institutions focus on the understanding of the subject which is the main goal of instruction, thus brought to the aim of this study – to analyze student understanding through examining their self-made questions. The study revolves in an assumption that understanding could be represented on how students make their own questions. To satisfy the purpose of this study, by examining self-made questions, the researchers sought to find out how students understand the concepts of linear equations, and the common misconceptions of students in linear equations. Qualitative content analysis is used to 20 self-made questions to identify the types of understanding and kinds of misunderstanding students showed in their questions. Results showed that three types of understanding were evident in students' self-made questions namely translate, extend, and judge. Students did not show idea type of understanding in their self-made questions. Consequently, six misconceptions were shown in students' self-made questions: choices present unnecessary details, choices present vague responses, choices do not answer the question, problem given does not present a question, questions present incomplete given, and intercepts are not coordinates. Analysis of the self-made questions reached to following conclusions that students have various forms of understanding and misconceptions are more on the recognition of the main idea.

Index Terms— **Understanding, Translate, Extend, Judge, Ideas, Self-Made Questions, Misconceptions, Coding frame, Coding**

I. INTRODUCTION

Understanding in mathematics is the connection of mathematical concepts and the recognition of these different concepts as a whole idea. (Hiebert, & Carpenter, 1992; Michener, 1978; Lehman, 1977). Mathematical theories are result of interconnections of latter mathematical concepts, thus to think mathematics as a whole system rather than separate subjects is an indication of understanding. The degree of understanding depends on the number and strength of links between information an individual have; on the other hand, lack of understanding is the mere isolation of this information (Hubbard, 1997). The more ways mathematical concepts are interrelated to each other, the more mathematics is understood.

Understanding of mathematics could be interpreted as understanding mathematical statements or theories (Lehman, 1997). Evidences must be presented as proof if a learner understands of such statements, thus researches suggest a number of indicators as sign to this occurrence. The study considered four indicators namely (1) Translate, (2) Extend, (3) Judge, and (4) Ideas (Buxkamper & Hartfiel, 2003). (1) To translate is to express and receive a concept in different conditions. A learner could recognize a mathematical concept in its applications; he/she could relate the concept to other ideas. (2) To extend means extending or adjusting it, as well as filling any gaps in it. One manifestation of this is when a student could extend a mathematical concept to a general case. A learner could present a proof to justify a certain concept; inversely, he/she could also use the concept to prove other ideas. (3) To judge is to make decisions on a concept. A learner who understands a concept could decide on the accuracy of a statement. (4) In Idea, a learner, from his/her prior knowledge, could give new ideas. A learner could combine concept to present a new concept, either in form of a proof or solution to a problem.

Understanding is the appropriate connection of knowledge; on the other hand, when such connections inaccurately connected, then misconception occurs (Ashlock, 2010). When a learner understands, then he/she forms framework in his/her mind; but in a Misconception, a learner forms an imperfect cognitive structure caused by erroneous knowledge (Mestre, 1989). It is necessary that misconceptions are corrected since it prevents accumulation of new learning, thus resulting to more misconceptions (Gilbert, 1982; Mestre 1989). Misconceptions are harmful to the learning process because learners tend to believe these concepts more, even though it is wrong, than to accept new correct concepts. It is hard to remove because false concepts may be deeply ingrained in the mental map of a learner, and their subjective nature hinders them to accept that what they know is incorrect (Li, 2006; Mestre 1989).

Promoting student understanding is the main purpose of mathematics instruction (Jones & Vermette, 2009; Hubbard, 1997; Hiebert, 1997). It is important that students not only learn math, they must learn it well due to the following reasons. First, learning can happen only by relating the unknown to what is already known (Skemp, 1987). This implies that learning could only gain by understanding since it is identified that to understand is to connect knowledge with each other to come up with new knowledge. Second, the 21st century offers new opportunities due to the emergence of new ideas and technology; on the other hand it also offers new challenges that require a new set of skills to compensate with these changes. The mere fact of knowing how to process mathematical problems such as solving equations is not enough due today's demand, a student must also sense of these concepts through analysis and synthesis of evidence (21st century skills, 2000).

Creating learning situations that demonstrate understanding among learners is indispensable (Hubbard, 1997; Hiebert, 1997). However, usual practice in Mathematics instruction consists of very restricted types of exercises and as a result, the students construct very restricted mental representations of the concept or procedure and fail to construct links to other knowledge (Hubbard, 1997). Classroom activities should be structured around problems, questions, and situations that may not have one correct answer to promote conceptual understanding (Wilson, 1996). Instruction should provide activities that include reflection and communication. Reflection is central for individual cognition and communication is central for social cognition. Communication works together with reflection to produce new relationships and connections, thus promote understanding (Hiebert et.al, 1997).

The study aimed to investigate the learners' conception and misconception. The researchers analyzed learners' self-made questions that demonstrate understanding, or in assessments the "understanding" level. From the analysis, we could gather data on what are their expectations and perceptions on understanding questions, thus we will identify which among these perceptions were correct and not.

The main purpose of the study is to investigate understanding of students on linear equations through analysis of self-made questions. The study sought to find out how students understand concepts of linear equation, and the common misunderstanding of students in linear equation.

II. CONCEPTUAL FRAMEWORK:

Understanding Mathematics in Construction of Questions

The individuals' understanding of their knowledge, learning preferences, styles, strengths, and limitations can determine how much they can perform on different tasks (de Carvalho, Magno, Lajom, Bunagan, & Regodon, 2006). Modern education perspective explains that students become aware of their own learning and eventually control their learning process which leads to better performance. Given this viewpoint, the main premise of this study was to show that it is possible for students to construct questions which aim to improve their own understanding in mathematics specifically in systems of linear equations.

Mathematics involves solving simple equations to complex ones. Mathematics is a field claimed to be not only restricted to solving problems with the use of complicated formula, but a springboard on how one must think and apply what one has learned to real life (Aquino, et al., 2003). Mathematics is also a field that determines the success and failure rates of the students depending on the learning strategy they utilized. Garofalo (1985) stated that the problem with some students was that when it comes to mathematics, they consider that certain problems are unsolvable if they are not competent to detect a solution for the problem at once. In mathematical problem solving, one needs the application of several cognitive skills such as identifying the elements, computing, analyzing the problem, synthesizing, and evaluating.

Students lack conceptual understanding if they memorize only a few facts, formulas, and algorithms without understanding them conceptually, even though they could manipulate those limited number of facts in a correct or incorrect manner (Erdoğan et.al, 2014). A different way to emphasize the links between concepts is to ask questions which involve the students to reverse their thinking. A very simple way to do this is to ask the student to make a question instead of answering it (Hubbard, 1997). As shown on the diagram (see Fig. 1), the understanding of student is observed through self-made questions.

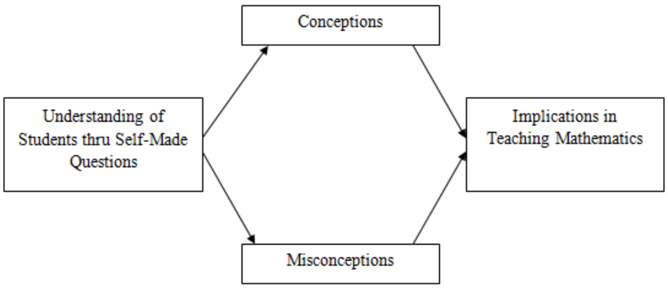


Fig. 1. Conceptual Framework

Conceptions recognize and relate factors that students use to clarify stimulating or challenging phenomena. They also characterize the knowledge, expressed in terms of solution strategies and their rationale that constitutes the core solution to specific problems. Change in how one makes a decision in favor of one conception over another is a complicated part of conceptual development (Kuhn & Phelps, 1982; Schauble, 1990). Understanding and skills can and should develop together but the major goal of mathematics instruction is conceptual understanding (Hiebert et al. 1997). This process is shown by the direction of the arrow from student-made questions towards conceptions.

In the next level of the diagram, the direction of the arrow shows that students have conceptions as well as misconceptions. Hasan, et al. (1999) claim, "Misconceptions are strongly held cognitive structures that are different from the accepted understanding in a field and that are presumed to interfere with the acquisition of new knowledge". Misconceptions originated from problems due to conceptual misunderstandings. Mistakes are derived from computational or minor mishaps (Ashlock, 2010). Misconceptions are characteristic of preliminary phases of learning because students' existing knowledge is insufficient and bears only partial understandings (Smith et al., 1993).

Students articulate their unconscious misconceptions and then establish a framework for evaluating the validity of the contending ideas (Champagne, Gunstone, & Klopfer, 1985; Strike & Posner, 1985). Consecutively to relate new ideas to existing ones the student needs to be involved in activities which aid in the organization building process (Hubbard, 1997). Students learn new topics by combining new knowledge with their preliminary knowledge. Thus, teaching activities should be planned by considering the knowledge and misconceptions of students. For that, the existing knowledge and the misconceptions (if any) of students should be determined (Gilbert, Osborne, and Fensham, 1982). This brings us to the last part of the diagram, the implications to education through investigation of student-made questions by understanding students' conceptions and misconceptions.

Vigorous classroom discussions are necessary in which students take positions, make sense of and explain problematic phenomena, and articulate justifications for their ideas. Activities that produce states of cognitive conflict are certainly desirable and conducive to conceptual change (Bereiter, 1985). Teachers play a vital role in lessening or eliminating the misconceptions held by students. The misconceptions of students should be determined before they lead to any mistake in the learning of subjects to be covered in the future. Research on student conceptions and misconceptions is a way to provide support for both teachers and students.

III. METHODOLOGY

As purpose of this study is to describe students' understanding on linear equations by analyzing selected self-made questions, the research method selected for this paper is qualitative content analysis (Schreier, 2014). This method was used to analyze textual data by giving meaning to students' self-made questions (Forman and Damschroder, 2008). Through qualitative content analysis, the study sought for common conceptions and misconceptions by reducing data from self-made questions to categories (Schreier, 2014).

Selecting Self-Made Questions

The study made use of 20 self-made multiple type questions with "understanding" level of assessment as data for analysis. Other features of the questions such as length of the question, number of choices, and construction of statements were under the discretion of the students. The questions were made by Private School students as part of their performance task requirements. Researchers asked permission to students to include their self-made questions in this paper. Questions were chosen considering a number of factors. First is availability; students should first agree to use their question for research purpose. Second is the type of question; students originally made 5 self made question in which one question is under knowledge level, two are under process, and two under understanding. The study only considered understanding questions as students perceived it. Third is the variety; researchers chose different sets of questions that have similarity with each other until reaching saturation. The purpose is to see a trend between the questions for the researchers to gather meaningful data.

Building a Coding Frame

A coding frame provides the classification system for the analysis of qualitative data; the self-made questions (Forman & Damschroder, 208). It consists of a main category and subcategories (Schreiner, 2014). The Main categories of the study were the four types of understanding: transfer, extend, judge, and idea. The subcategories are the purpose of the question; what does the question asked.

The Main categories are defined in this study according Buxkemper & Hartfiel (2003) study on Understanding Mathematics. Each category is based on the definitions of the four types understanding.

(1) Translate—be able to put, and receive, material in forms other than that originally presented. A student should also be able to do something with material they understand.

(2) Extend—be able to extend material, including filling in missing parts.

(3) Judge—be able to make conclusions based on correctness as well as by comparing and contrasting.

(4) Ideas—get ideas about how material is, or can be, put together to form a whole, and why it works.

The definitions are used to categorize the questions; however, further discussions that relate to the four definitions were also considered.

The subcategories grouped the questions according to its suggested purpose. What is asked in each questions were collected to identify the purpose of each self-made questions. The study, with accordance of the questions, did able to form a trend of purposes, (1) Check solution, (2) Problem solving, (3) Matching, (4) Graphing. Each purpose is classified and described according to what is asked in the problem, also with the structure and skills an individual need to answer the question.

(1) Check Solution – the question asks if a process is correct or not.

(2) Problem Solving – the questions asks to translate and solve a word problem.

(3) Matching – the question asks to check which of the following should be paired or which of the following should be excluded.

(4) Graphing – the question involves analyzing graphs and other interpretation of it.

Coding on Understanding

The 20 self-made questions are classified according to the definitions and descriptions of the Main categories and subcategories. Coding of subcategories was done first before grouping the subcategories to main categories. In subcategories, self-made questions were classified into three categories: questions, asked in each question, purpose of each question. Self-Made questions were categorized according to type of understanding so that the researchers could identify how students understand linear equations. Each type of understanding (transfer, extend, judge, and idea) are analyzed by looking for evidence that students possess them.

Coding for Misconceptions

From the 20 self-made questions, questions with errors are identified. For each erroneous questions are analyzed further resulting to identify the specific parts that made the question erroneous. Incorrect parts of the questions were coded depends on the error each commit. Categories used in coding were described depends on the type of error, thus misconception it represents. Each type of misconception is analyzed to define what kind of misconceptions the each self-made question represents. Codes are used to categorize errors that represent the same misconception. After coding, each type of misconceptions are analyzed further to define specific indicators how the question became erroneous.

IV. RESULTS

How students understand the concepts of linear equations

The study revolves at an idea that self-made questions is a good evidence for a student's understanding of a topic, which in this paper, is linear equations. The data used in this research consists of 20 student self-made questions. These questions underwent coding to two set of categories, the main categories and the subcategories, which are further explained in the methodology. The table below shows the coding subcategories (see table 1).

TABLE 1: SUBCATEGORIES (PURPOSE OF THE QUESTION)

Subcategories (purpose of question)	Question	What is asked in the question
Check Solution	Question 1	*Who is correct and why?*
Check solution, problem solving	Question 2	*Is Clark's Solution correct?*
	Question 3	*Are her computations correct?*
	Question 6	*Which of the following will give the correct present age of the two brothers?*
	Question 7	*Is Jacob's solution going to be correct?*
	Question 8	*Which of these solutions are correct?*
	Question 13	*Check if Sofia's solution and answer to the problem are correct:*
	Question 14	*Enzo solved for the question below, did he solve it correctly?*

	Question 15	*Is the final answer correct or incorrect? Why?*
	Question 16	*Solve the coordinate to find out which boat intersects Tanker Baba GH.*
	Question 17	*His teacher said he was incorrect. Why was he wrong?*
Check solution, graphing	Question 4	*Did she graph it correctly? Why or why not?*
Problem Solving	Question 5	*In how many years will Red be an adult?*
	Question 9	*Will they be able to afford the airline fees with their limited budget?*
	Question 10	*Who will catch it?*
	Question 18	*Will the frog be able to catch the fly? Why or why not?*
	Question 19	*Find their present ages.*
Matching and Graphing	Question 11	*What way is different from the rest?*
	Question 12	*Is the table of values and the graph displaying the same relationship between the independent variable "x" and the dependent variable "y"?*
	Question 20	*Which solution from the equation x = 2y + 8 matches the graph?*

Table 1 shows the purpose of each question considering *"what is asked"* and *"construction of question"*. Most common purpose suggested by the students is to check the solution of a problem; specifically, solutions for word problems. The usual set up is a word problem with solution will be given and then asks if the solution is correct. In case of graphing linear equations, students tend to match various representations (equation, table of values, graphing). Students usually asked if an equation, table of values, or graph represents each other or which of the three does not represents the other. By analyzing the questions depending on their purpose give the researchers 5 subcategories: (1) Check Solution, (2) Check solution & problem solving, (3) Check solution & graphing, (4) Problem solving, (5) Matching & graphing.

The five subcategories further grouped to four main categories that represents the four types of understanding namely; transfer, extend, judge, and idea (Buxkemper and Hertfiel, 2003). Decisions in classifying questions depending on the type of understanding it represents were based on the definition the literature suggested. The five subcategories were used to list indicators for categorizing the questions into types of understanding (see Methodology). Subcategories (2) Check solution & problem solving, (3) Problem solving are categorized under transfer. (1) Check Solution, (2) Check solution & problem solving, (3) Check solution & graphing are categorized under judge. Extend and idea categories are categorized according to further analysis of the questions which will be shown later in this chapter.

Questions are categorized into four types of understanding; however, coding of questions suggests other categories. These categories are interpreted as questions that show two types of understanding, thus questions are categorized into six (see Table 2). Data showed that a student could represent combinations of understanding in a single question.

Questions are categorized depending on the type of understanding it represents; however questions have combination of subcategories which require further analysis. First; how extend and idea type of understanding is categorized. Second; how different categories are combined in a single self-made question.

TABLE 2: MAIN CATEGORIES (TYPE OF UNDERSTANDING)

Main categories (type of understanding)	Question	Subcategories (purpose of questions)
Transfer	Question 5	Problem solving
	Question 6	Check solution, problem solving
	Question 8	Check solution, problem solving
	Question 10	Problem solving
	Question 16	Check solution, problem solving
	Question 18	Problem solving
	Question 19	Problem solving
Extend	Question 11	Matching, graphing

	Question 20	Matching, graphing
Judge	Question 1	Check solution
	Question 12	Matching, graphing
Translate and Extend	Question 9	Problem solving
Translate Judge	Question 2	Check solution, problem solving
	Question 3	Check solution, problem solving
	Question 7	Check solution, problem solving
	Question 13	Check solution, problem solving
	Question 14	Check solution, problem solving
	Question 15	Check solution, problem solving
	Question 17	Check solution, problem solving
Extend and Judge	Question 4	Check solution, graphing

Translate

From the set of self-made questions, 16 questions are identified to be evidences that students could translate concepts of linear equations to other ideas. To translate means one is able to put and receive a concept other than originally presented (Buxkemper and Hertfiel, 2003). Self-made questions having able students to translate tend to be in two types: First, linear equations are in form of an applied word problem. Second, linear equations are shown in different representations.

The first type, in which applied to word problem was shown in question 3 (see fig. 2). The self-made question is an age problem.

> *"Four years ago, Rosie was thrice as old as her son, Alex. Five years later, she was twice as old. Find their present age."*

Fig. 2. Question 3

The problem tests if the examinee could translate the statements of the problem into a system of linear equation to find the age of Rosie and Alex. Thus, the self-made question also showed that the students' able to present linear equations other than in expression form. Statements, according to the students' suggested solutions, are presented into two equations; (1) $x - 4 = 3y - 12$ and (2) $x + 5 = 2y + 10$.

Other way students presented a word problem was shown in question 10 (see fig. 3).

> *"At the softball game, Hazel hit a line drive with a flight of $2x + 4y = 8$. If Alex, Jolo and Riel are fielders who are aiming to catch the ball, who will catch it? Alex is running on the path $4x + 8y = 1$, while Jolo is running on the path $12x - 20y = 4$, and Riel on the path $6x + 12y = 6$. Assume that they can all catch the ball as long as they meet it."*

Fig. 3. Question 10

The problem, just like in question 3, also asked the examinee to use system of linear equation to solve the problem; however, this problem was different in a way that statements are not translated to equations. Equations are used to represent pathways and directions. Students tend to see linear equations as linear paths that could intersect to each other which could be determined if you solve system of linear equations. This problem shows transfer since you first need to see linear equations as lines then check if the lines intersect.

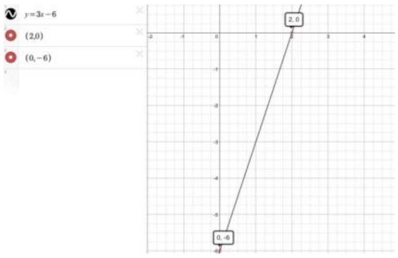
The second type was transfer in a sense that system of linear equations are presented into a Cartesian plane as a line. It was shown in question 11 (see fig. 4).

"There are three ways to represent x and y relationship (table of values, graph, equation).One of the three ways is different from the other two. What way is different from the rest?"

EQUATION: y=3x-6

TABLE OF VALUES:

x	3x-6
1	-3
6	12
5	10
3	3



a) Equation *c.) Graph*
b) Table of Values *d.) None of the Above*

Fig. 4. Question 11

The examinee is asked to choose which one of the three representations of linear relationship does not represent the other two. They should see if one of the representations does not match the others. This is another evidence of transfer different from using word problems. Transfer means one can represent a concept to other forms, the question which checks different representation of linear equations is a perfect fit.

Extend

To extend a concept means able to fill the missing parts of the said concept (Buxkemper and Hertfiel, 2003). Only 4 self-made questions were qualified in this type of understanding, one of it is question 9 (see fig. 5).

"Fides and her dad Alex are trying to book a flight to Singapore. While looking through the travel fees, they found out that the airline offers a 20% discount to minors (below 18 years old). They have a budget of P15,000.00 and each ticket costs P8,000.00. Will Fides get the discount if her dad is 3 times as old as her, and if he will be twice as old 15 years from now? Will they be able to afford the airline fees with their limited budget?"

Fig. 5. Question 9

The question was evidence that a student could extend the idea of two kinds of problems: money, and age problems. The problem started with given for money problem, as stated *"the airline offers a 20% discount to minors (below 18 years old). They had a budget of P15,000.00 and each ticket costs P8,000.00"*. The given showed that Fides and Alex have a budget of only P15,000.00 with ticket rides of P8,000 each, To answer the problem one must first answer if Fides and Alex could afford the two tickets in any of the cases. There are three cases, first is none of them will have a discount, second is only one of them will have a discount, and third is both of them will have a discount. This is already a form of understanding by extent because an examinee needs to fill out the cases needed to be considered in order to determine if it is really possible for Fides and Alex to afford the tickets.

Continuing with the cases, the first case will definitely not be a possible scenario for Fides and Alex to afford the ticket since if the two buy a ticket worth P8,000 each, they will reach an amount of P16,000. For the second case, one must first compute for the combined for the total price of the tickets. If Fides could avail the discount, then the total price will be P14,400 which is under their budget. If the second case could be afforded by the two, then the third case will be affordable also.

Since the discount is for minors (18 years old below), the students made another set of given *"her dad is 3 times as old as her, and if he will be twice as old 15 years from now"*. This is a form of extent because the question makes a connection from money problem to an age problem. The given will serve as a proof if Fides is a minor or not, which is an important in determining if she and Alex could avail the tickets. An examinee could answer the problem in two steps: First, the three cases shown earlier should be determined. Second, the case that will be considered should be decided depending on the computation of Fides' and Alex's ages.

Judge

A student could judge if he/she could make conclusions on the correctness of an idea (Buxkemper and Hertfiel, 2003). 12 of the questions could test if an examinee could judge a certain concept. By analyzing the self-made questions, the students' common conception of judging observed is either asking the correctness of a given solution or selecting among the solutions the correct one.

The first format, correctness of a given solution, was the more common type of judging type of question. The usual format of students' questions starts with a problem, either simple solving or word problems. Then the question will provide a complete solution that answers the problem. The question was usually *"Is the solution correct?"* and then followed by *"Why?"*. This format of question could be represented by question 2 (see fig. 6).

The question was a clear example of judging because the main question is "Is Clark's solution correct?". The question itself directly asks the examiner to check if the solution is correct or not. Other evidence that the question was meant for judging is the construction of its choices.

Choices were divided into two main answers, correct or not; however, choices were further divided according to possible reasons for being correct or not. There was only one choice which suggests that the solution is correct, thus the question suggests only one reason for this "the solution is complete and the final answer is correct". On the other hand, the incorrect answers provide different reasons. If the choice that suggests a correct solution is not the answer, then the answer is among the three choices that suggests an incorrect solution. One of these reasons is correct while the other two are distracters. These choices tend to make the examinee check the provided solution, and then they will decide which part of the solution is incorrect. The reason why the solution is incorrect may come from an intended wrong part of the provided solution.

"Clark is trying to find the ages of his two siblings. Half of Lilia's age added to a third of Dominic's age gives a result of 16. In two years, the sum of their ages is 42. Is Clark's solution correct?"

Let x be Lilia, and y be Dominic.

Equation 1: $\frac{1}{2}x + \frac{1}{3}y = 16$

Equation 2: $x + y + 2 = 42$

$x + y = 40$

$x = 40 - y$

$6\left(\frac{1}{2}x + \frac{1}{3}y\right) = (16)6$

$3x + 2y = 96$

Substituting $x = 40 - y$ into $3x + 2y = 96$,

$120 - 3y + 2y = 96$

$y = 29$

$x + 26 = 40$

$x = 16, y = 29$

Therefore, Lilia is 16 years old, and Dominic is 29 years old.

a) Correct, because his solution is complete and his final answer is correct

b) Incorrect, because his solution to the system of linear equation is incorrect

c) Incorrect, because he incorrectly translated a statement into an equation

d) Incorrect, because his answer only solves one equation, not both

Fig. 6. Question 2

The second format is where the examinee will select from a set of solutions is correct to solve the problem. The format is that the question will start with a question, usually a word problem. Next is that each choice will provide a possible solution for you to choose. This format is shown in question 6 (see fig. 7).

"Mark, a government official was tasked to find out the ages of Josh and his younger brother Gino who is 20 years younger. In 2 years, Josh will be twice as old as Gino. Which of the following will give the correct present age of the two brothers?"

a. Let x = Josh's present age Let y = Gino's present age
y=x-20
x+2=2(y+2)
x+2=2(x-20)
x+2=2(x-20)
x+2=2x-40
-x=-40
x=40 y=40-20
y=20
Therefore Gino = 20 years old and Josh = 40 years old

b. Let x = Josh's present age Let y = Gino's present age
y=x-20
x+2=2(x+2)
x+2=2(x+2)
x+2=2(x+2)
x+2=2x+4
-x=2
x=-2 y=-2
y=-22
Therefore Gino =22 years old and Josh = 2 years old

c. Let x = Josh's present age Let y = Gino's present age
y=x-20
x+2=2(y+2)
x+2=2(x-20+2)
x+2=2(x-18)
x+2=2x-36
-x=-38
x=38 y=38-20
y=18
Therefore Gino = 18 years old and Josh = 38 years old

d. Let x = Josh's present age Let y = Gino's present age
y=x-20
x+2=2(x+2)
x+2=2(x+2+2)
x+2=2(x+2+2)
x+2=2x+8
-x=6
x=-6 y=-6-20
y=-26
Therefore Gino =26 years old and Josh = 6 years old

Fig. 7. Question 6

What are the common misconceptions of students in linear equations?

Included in the purpose of the study was to identify misconceptions presented in the problem. 11 questions showed significant misconceptions; while six types of misconceptions were identified (see table 3). Misconceptions shown are (1) Choices present unnecessary details, (2) Choices present vague responses, (3) Choices do not answer the question, (4) Problem given does not present a question, (5) Questions present incomplete given, and (6) Intercepts are not coordinates. Most of the misconceptions were not subject based; meaning mistakes came from the construction of the question and not from erroneous mathematical concepts.

Choices Present Unnecessary Details

Questions were in form of multiple choices, thus a student should present choices that represent different possible responses from examinees. The first type of misconception concerns with unnecessary details on the responses. To further discuss this point, question 6 is used an example (see fig. 8).

> *"Mark, a government official was tasked to find out the ages of Josh and his younger brother Gino who is 20 years younger. In 2 years, Josh will be twice as old as Gino. Which of the following will give the correct present age of the two brothers?"*

Fig. 8. Question 6

The question asks if which of the following solution will give a correct answer to problem, thus the problem wanted to check if the examinee knows how to evaluate a solution. In the problem, the solutions are presented thru the four choices (see fig. 9).

a. Let x = Josh's present age   Let y = Gino's present age
y=x-20
x+2=2(y+2)   x+2=2(x-20)
x+2=2(x-20)
x+2=2x-40
-x=-40
x=40   y=40-20
y=20
Therefore Gino = 20 years old and Josh = 40 years old

b. Let x = Josh's present age   Let y = Gino's present age
y=x-20
x+2=2(x+2)   x+2=2(x+2)
x+2=2(x+2)
x+2=2x+4
-x=2
x=-2   y=-2
y=-22
Therefore Gino =22 years old and Josh = 2 years old

c. Let x = Josh's present age   Let y = Gino's present age
y=x-20
x+2=2(y+2)   x+2=2(x-20+2)
x+2=2(x-18)
x+2=2x-36
-x=-38
x=38   y=38-20
y=18
Therefore Gino = 18 years old and Josh = 38 years old

d. Let x = Josh's present age   Let y = Gino's present age
y=x-20
x+2=2(x+2)   x+2=2(x+2+2)
x+2=2(x+2+2)
x+2=2x+8
-x=6
x=-6   y=-6-20
y=-26
Therefore Gino =26 years old and Josh = 6 years old

Fig. 9 Choices for Question 6

The style of showing the questions in the choices is not erroneous. It's also not a problem if each choice presents different solutions; however, having different answers made the question erroneous. If the choices present same answers, then the question would check if the examinee knows how to correctly come up with the answers. In the case of question 6 with different final answers, the examinee will just answer the age problem and then see which of the choices have the same answer as he/she has. Therefore there is no need to look for the solution in the first place, making the choices present a redundant detail.

TABLE 3: MISCONCEPTIONS

Coding (Misconception)	Question	Incorrect part of the questions
Choices present unnecessary details	Question 1	*Choices:* *A: Jeremy is correct because* $\frac{17}{2}-10$ *is equal to* $\frac{3}{2}$ *making y equal to 6 and x equal to 8.* *B: Sage is correct because* $\frac{17}{2}-10$ *is equal to negative* $\frac{3}{2}$ *making y equal to -6 and x equal to 32.*

	Question 6	*C: Both of them are wrong because they did not substitute properly.* *Choices (one choice presented)* a. Let x = Josh's present age Let y = Gino's present age y=x-20 x+2=2(y+2) x+2=2(x-20) x+2=2(x-20) x+2=2x-40 -x=-40 x=40 y=40-20 y=20 Therefore Gino = 20 years old and Josh = 40 years old
	Question 17	*Choices* *a. 1st equation he made was wrong, this in turn made the substitution method and his solution wrong. Since he got one part of his solution wrong, he is now solving for something not asked for in the question.* *b. 2nd equation was wrong, this in turn made the substitution method and his solution wrong. Since he got one part of his solution wrong, he is now solving for something not asked for in the question*
Choices present vague responses	Question 3	*Choices:* *a) Correct and complete* *c) Incorrect but complete* *b) Correct but incomplete* *d) Incorrect and Incomplete*
	Question 4	*Choices:* *II. She didn't change the sign when she transposed* *III. She substituted the wrong value* *IV. She graphed incorrectly.*
	Question 13	*Choices:* *b) Wrong, because she distributed it wrong* *d) Wrong, because she divided wrong*
	Question 14	*Choices:* *c) Incorrect because there was a fault in the solution* *d) Incorrect because he understood the problem wrong*
Choices does not answer the question	Question 12	*Choices:* *c) None of the Above*
	Question 15	*Choices:* *d.) no solution*
Problem given did not present a question	Question 15	*The sum of the husband's age and the wife's age is 74. In four years, the husband will be 2 years older than the wife. Is the final answer correct or incorrect? Why?*
Question Present Incomplete Given	Question 16	*Tanker Baba GH at point (2, -5) is running out of gas. There are 3 other tankers such as Brago AB at (-4, 5), JME EF at (3, -3), Sobs CD (-2, 3). Solve the coordinate to find out which boat intersects Tanker Baba GH.*
Intercepts are Not Coordinates	Question 20	Solution A: $0 = 2y + 8$ $\frac{-2y}{-2} = \frac{8}{-2}$ $\frac{-\cancel{2}y}{-\cancel{2}} = -\frac{8}{2}$ $y = -4$ Solution B: $0 = 2y - 8$ $\frac{-2y}{-2} = \frac{-8}{-2}$ $y = 4$ $x = 2(0) + 8$ $x = 0 + 8$ $x = 8$ $x = 2(0) - 8$ $x = 0 - 8$ $x = -8$

Choices Present Vague Responses

Most of the misconceptions in the questions are caused by providing unspecific details to choices. To discuss this point, question 14 (see fig. 10) is served as an example. The question shows the equation that shows Enzo's solution. The main purpose of the problem is to check if his solution is correct. In response with the question, the students presented possible responses. One choice shows correct answer while the other three choices show incorrect answers. Incorrect answers provide three different reasons why the solution (Enzo's solution) is incorrect.

Enzo solved for the question below, did he solve it correctly?

The larger number is 3 more than twice the smaller number. If three times the larger number is subtracted from twice the smaller number, the answer is -45.

Here is how Enzo solved for the problem: x = larger number y = smaller number Equations: x = 2y + 3 2y - 3x = -45	Solve: 2y - 3(2y+3) = -45 2y - 6y - 9 = - 45 4y = -36 y = -9 x = 2(-9) + 3 x = -18 + 3 x = -15 The final answer is (-15, -9)

a. Incorrect because he used the wrong equation

b. Correct because all solutions were correct

c. Incorrect because there was a fault in the solution

d. Incorrect because he understood the problem wrong

Fig. 10. Question 14

Choices C and D are presented vaguely for certain reasons. For choice C *"Incorrect because there was a fault in the solution"*, the choice is vague in a manner the stating that the solution is faulty, is too broad. The whole process of solving the problem is the solution. The response should be more specific. The choice must show which part of the solution is faulty. In the case of choice D *"Incorrect because he understood the problem wrong"*, the choice is vague because how could an examinee know that Enzo understood the problem wrong. The choice should focus on specific indicators that could be shown concretely.

Choices Do Not Answer the Question

Choices are possible answer to the question, thus at least it could comply with what is asked in the problem. Some of the questions presented choices that did not answer the question at all such as question 12 (see fig. 11).

Is the table of values and the graph displaying the same relationship between the independent variable "x" and the dependent variable "y"?

a) Yes, because equation x = 0 represents the y-axis

b) No, because equation x = 0 represents the x-axis

c) None of the Above

Fig. 11. Question 12

Choice A and B were correct; however, choice C does not answer the question at all. As shown in the question "*Is the table of values and the graph displaying the same relationship"*, the question expects a YES or NO question. A response of "*None of the Above"* is inappropriate because if the answer in the question is not YES, then it is automatically NO. If the answer is not NO, then it is a YES. The question does not allow any responses between YES or NO because it is not scaled how YES or how NO the answer is.

Problem Given Does Not Present a Question

Many of the questions used problem solving, which requires a specific question. Question 15 is an example that shows a misconception that some problems do not have a specific question (see fig. 12).

"Jessica was answering a test paper and she got stuck at number 8. The question says:
The sum of the husband's age and the wife's age is 74. In four years, the husband will be 2 years older than the wife.
Is the final answer correct or incorrect? Why?"

Fig. 12. Question 15

The question's purpose is to check if the solution in the given problem is correct. It is clearly shown in the question by having the questions "*Is the final answer correct or incorrect? Why?"*. On the other hand, the age problem only shows given but did not ask what really are you going to find. If we check the problem *"The sum of the husband's age and the wife's age is 74. In four years, the husband will be 2 years older than the wife"*. You have the statements that could be translated to equations; however, the problem does not have a question. It does not ask what are to look for, is it the age of the husband or the age of the wife.

Questions Present Incomplete Given

Missing given could cause misconceptions in a question. The problem in question 16 is an example of a problem with missing given (see fig. 13).

"Tanker Baba GH at point (2, -5) is running out of gas. There are 3 other tankers such as Brago AB at (-4, 5), JME EF at (3, -3), Sobs CD (-2, 3). Solve the coordinate to find out which boat intersects Tanker Baba GH."

Fig. 13. Problem in Question 16

The question is to "solve the coordinate to find out which boat intersects Tanker Baba GH". The error in this question is that there is no specific given if Tanker Baba GH is moving or not, the only given is that it is running out of gas; the same with the other three ships. If the examinees are going to take the problem as it is, all the ships are stationary; but, the information that they are not stationary is not giving thus assuming it will be inappropriate. The problem itself causes confusion, added the choice that none of the above will intersect Tanker Baba GH which gave an option that the four ships are stationary.

Intercepts Are Not Coordinates

Intercepts are points in a function that intersects the x-axis and the y-axis, thus intercepts must have x and y coordinates. Some of the questions showed a problem that intercepts are not in intercept form, an example of this question 20 (see fig. 14).

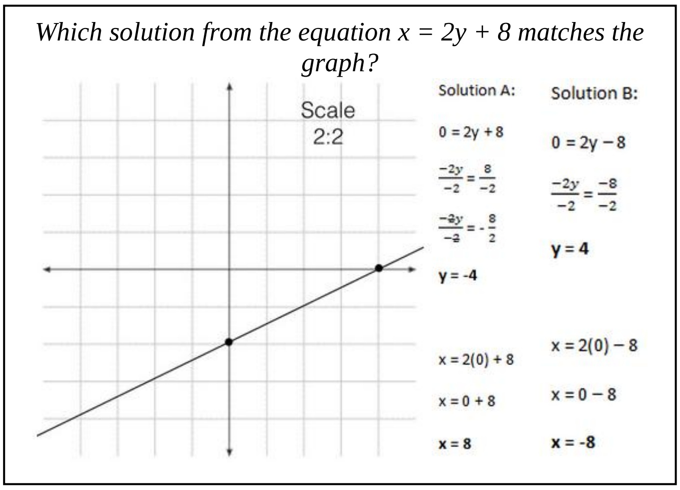


Fig. 14. Question 20

In this question intercepts are not written in coordinates. Y intercepts are written in y = -4 and y = 4, which should be written as (0, -4) and (0, -4). Same with the x intercepts where x = 8 and x = -8, which is supposedly written in (8, 0) and (-8, 0).

VII. SUMMARY

Summary

By qualitatively analyzing the self-made question, the researches come up with the following results:

1. Students showed different kind of understanding thru making self-made questions. Three types of understanding were evident namely (1) translate, (2) extend, and (3) judge. Students did not able to show (4) idea type of understanding in their self-made questions.
   a) Students presented two formats in presenting questions that shows translate type of question. First, linear equations are in form of an applied word problem. Second, linear equations are shown in different representations.
   b) Students presented two formats in presenting questions that shows judge type of question. First is asking the correctness of a given solution. Second is selecting among the solutions the correct one.
2. Most of students' misconceptions are not subject based. Mistakes came from the construction of the question and not from erroneous mathematical concepts.
3. Misconceptions shown by students' self-made questions are the following: (1) Choices present unnecessary details, (2) Choices present vague responses, (3) Choices do not answer the question, (4) Problem given does not present a question, (5) Questions present incomplete given, and (6) Intercepts are not coordinates.

Conclusions

On the basis of the findings, the researcher came up with the conclusions on the implications of the study to instruction of Mathematics:

1. Students have their different versions of understanding in a certain concept. The construction of different formats of questions is an indicator that understanding has various forms, thus instruction must comply with this variability.
2. Students' misconception is not on the process of solving; instead, it is more on the recognition of the main idea in a problem. Inadequate information of questions could lead to a fact that students lack skills in identifying important aspects of a problem.

Recommendations

Moreover, in the light of the findings of this study, for future researchers who will be interested in further continuing or improving the study, the researcher further recommend:

1. Use more self-made questions for analysis to observe a more diverse and meaningful data about conceptions and misconceptions of students regarding linear equations.
2. Develop innovative teaching approaches that could practice students' different forms of understanding of a certain concept in Mathematics.
3. Explore strategies on how to identify and remedy students' misconceptions in Mathematics.

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