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DESIGNING OPTIMAL CONTROL OF CONSTRAINED DC-DC POWER CONVERTER USING PARTICLE SWARM OPTIMIZATION

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Abstract- In this paper, a new algorithm which is a combination of model predictive control with particle swarm optimization is presented to optimal control of constrained DC-DC power system modeled as piecewise affine. Two problems are being addressed: one is deriving the control law of constrained final time optimal control for DC_DC power system based on model predictive control over polyhedral regions and the other is using particle swarm optimization method to reduce the number of polyhedral and improve DC-DC performance simultaneously. Simulation results demonstrate the potential advantages of the proposed methodology and illustrate that how the complexity of optimal control law can be efficiently reduced along with improvement of DC-DC performance using particle swarm optimization.

Keywords- Particle swarm optimization, Complexity reduction, Constrained final time optimal control, Piecewise affine system, DC-DC power system.

I. Introduction

In the last few years, many interests have been aroused in the synthesis of hybrid systems, especially Piecewise Affine (PWA) (sontag 1981) systems. One of the most significant subject regarding the constrained PWA systems is computation of closed-form optimal control law. This issue is famous as Constrained Final Time Optimal Control (CFTOC) problem (Borreli 2003), (Christofersen 2007).

The major proposed algorithm to solve the CFTOC is Model Predictive Control (MPC) (Maciejowski 2001). The closed-form solution based on MPC is a time-varying PWA control law over polyhedral regions (Mayne et al.2000), (Borreli2003), (Lazar2006), (Christofersen 2007). Despite all advantageous properties such as guarantee of stability (Christofersen2007) and its straight-forward design procedure (Beccuti et al. 2008), this method has some disadvantages, as well. The uttermost of them is the online computational complexity of a solution that grows exponentially with increasing the number of polyhedral regions and the increment of size in the required memory. Several attempts have been made to reduce them based on reduction of polyhedral regions (Borreli 2003), (Christophersen 2007). In some studies, the abovementioned topic is the only discussed issue, albeit other ones such as stability and improvement of performance have not been paid much attention (Almer and Morari 2007).

Due to comprehensive nature of CFTOC of PWA systems and the extension of its related topic, a host of practical systems are modeled in the PWA form (Heemels and Bemporad 2001), (Borreli2003), (Christofersen et al.2003), (Stefan et al. 2010), (Kawashima et al 2012).

DC-DC power converter (Kassakian et al.1991) is a basic benchmark of the physical system that can be modeled by PWA form. To analyze the issues related to the PWA systems, many researchers have tackled constrained DC-DC converter optimal control problem through various approaches. Take Beccuti et al for example who has presented the optimal control of the boost DC-DC converter problem (2005). The regulation of an average output voltage to desired value with favorable output specification as fast as possible were the control objectives in this work. Vujanic in 2008 obtained the control law of PWA buck-boost converter based on MPC. The paper was based on the optimized balance between conduction and switching losses. Geyer et al introduced the hybrid model predictive control of the step-down DC–DC converter in 2008; consequently, adding a Kalman filter to achieve zero steady-state output voltage error was investigated. However, a major problem of the issue is computational complexity of the control law.

From our point of view, due to the complexity of CFTOC solution based on MPC strategy for DC-DC converter and its strong dependence on the number of polyhedral, the problem can be categorized to NP-hard problem. To solve the NP-hard problem, the evolutionary algorithms are much more fruitful than analytical methods. The emergence of new generation of powerful computers is led to increase the tendency of using evolutionary algorithms to solve the NP-hard optimization problems. The evolutionary algorithm is classified to heuristic, meta-heuristic and hyper-heuristic algorithm.

Hyper-heuristic algorithm is used to solve the NP-hard optimization problems that have strategies to escape from the local optimal solution and are applicable in broad range of issues. In general, the development of hyper-heuristic methods facilitated by investigating and inspiration optimization type in the nature such as Genetic Algorithms (GA) (Eberhat and Yuhui 1998), Ant Colony Optimization (ACO) (Dorigo et al.1996) and Particle Swarm Optimization (PSO)(Kennedy and Eberhartl.1995).

The considerable merits of PSO are easily implementation and the ability to optimize complex objective functions with a number of local minimums. In addition, PSO can conduct a search of much extended space of candidate solutions. More specifically, PSO does not utilize the gradient of the problem which is being optimized, which means PSO does not require that the optimization problem be differentiable as it is required by classic optimization methods such as gradient descent and Gaussi-Newton methods. This property may increase the PSO ability in solving the partially irregular, noisy problems which could be changed over time (Clerc and Kennedy2002).

Therefore, a new combined algorithm based on PSO is going to be applied in order to reduce the complexity of explicit MPC-based solution CFTOC of DC-DC converter; consequently, the number of polyhedral is minimized, system performance is improved and even the existing PID coefficients are optimized simultaneously.

The present paper organized as follows:

To start, CFTOC of PWA systems is expressed briefly. Having introduced PSO in section 3, a Buck-Boost DC-DC converter will be presented and then CFTOC problem will be expressed in section 4. The application of PSO to solve the presented problem is discussed in section 5, and eventually the simulation result and conclusion are going to be presented.

II. CONSTRAINED FINAL TIME OPTIMAL CONTROL OF PWA SYSTEMS AND SOLUTION

Piecewise affine systems presented themselves as powerful frameworks to form very general nonlinear systems (Sontag1981), (Borreli,2003).

The class of constrained PWA systems as Eq.(1) will be focused (Christofersen et al.2003):

$$x(t+1) = f_{PWA}\left(x(t), u(t)\right) := A_i x(t) + B_i u(t) + a_i \quad \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} \in D_i \qquad (1)$$

Where $t \geq 0$, $D := \cup_{i=1}^{N_D} D_i$ denotes a polyhedral partition of the domain D; D the domain of $f_{pwa}(.,.)$ that is a non-empty compact set in $R^{n_x+n_u}$ i.e. the closure of D_i is $\bar{D}_i := \left\{ \binom{x}{u} \in R^{n_x+n_u} \middle| D_i{}^x x + D_i{}^u u \leq D_i{}^0 \right\}$, $int(D_i) \cap int(D_j) = \emptyset \ \ \forall i \neq j$

; $N_D<\infty$ is the number of system dynamics.

The CFTOC problem for PWA system Eq.(1) in the form of Eq. (2.a-c) is defined (Christofersen et al.2003):

$$J_T^*\left(x(0)\right) := \min_{U_T} J_T\left(x(0), U_T\right) \qquad (2.a)$$

$$s.t \quad \begin{cases} x(t+1) = f_{PWA}\left(x(0), u(t)\right) \\ \qquad x(t) \in \chi^f \end{cases} \qquad (2.b)$$

$$J_T\left(x(0), U_T\right) := l_T\left(x(T)\right) + \sum_{t=0}^{T-1} l\left(x(t), u(t)\right) \qquad (2.c)$$

Where $J_T(.,.)$ is the cost function, l(.,.) the stage cost, $l_T(.)$ the final penalty, U_T optimization variable described as the input sequence $U_T := \{u(t)\}_{t=0}^{T-1}$, $T<\infty$ receding horizon and χ^f is a compact terminal target set in R_{nx}. If the solution of CFTOC problem is not unique, $u_T^*\left(x(0)\right) := \{u^*(t)\}_{t=0}^{T-1}$ determines one realization from the set of possible optimizer.

CFTOC problem determines a set of initial state and feasible inputs as $\chi_T \subset R^{n_x}(x(0) \in \chi_T)$, $U_{T-t} \subset R^{n_u}(u(t) \in U_{T-t}, t = 0, \dots, T-1)$ respectively. The aim of this section is to obtain an explicit closed-form statement for $u^*(t): \chi_T \rightarrow U_{T-t}$, t=0,…,T-1. The considered system is PWA of Eq. (1) and the cost based on 1, ∞ norm. i.e Eq. (3.a-b).

$$l\left(x(t), u(t)\right) := \|Qx(t)\|_p + \|Ru(t)\|_p \qquad (3.a)$$

$$l\left(x(T)\right) := \|Px(T)\|_p \qquad (3.b)$$

Where $\|.\|_p$ with p={1,∞} represent the standard vector norm 1,∞. The solution of optimal control described as Eq.(2) with aforementioned restrictions is time-varying PWA function of the initial state x(0):

$$u^*(t) = \mu_{PWA}\left(x(0), t\right) = K_{T-t,i}\, x(0) + L_{T-t,i} \quad \text{if } x(0) \in \mathcal{P}_i$$

Where $t = 0, \dots, T-$, $\{\mathcal{P}_i\}_{i=1}^{N_p}$ is the polyhedral partition of set of feasible state x(0), $\chi_T = \cup_{i=1}^{N_p} \mathcal{P}_i$ with the closure of $\mathcal{P}_i$ stated as $\bar{\mathcal{P}}_i = \{x \in R^{n_x} | \mathcal{P}_i^x x \leq \mathcal{P}_i^0\}$ (Christofersen et al.2003).

If a model predictive control strategy is used for closed loop, the control is stated as time-varying PWA state feedback of the form of Eq.(4) (Christofersen 2007):

$$\mu_{RH}\left(x(t)\right) := K_{T,i}\, x(t) + L_{T,i} \quad \text{if } x(t) \in \mathcal{P}_i \qquad (4)$$

Where i=1,…,N_D and for $t \geq 0$, $u(t) = \mu_{RH}\left(x(t)\right)$.

Now a brief description about PSO is presented to use in the next sections.

III. Particle Swarm Optimization

Particle swarm optimization is a hyper heuristic global optimization method stated originally by Kennedy and Eberhart in 1995. In actual fact PSO is a computational method that optimizes a problem by iteratively tries to improve a candidate solution with regard to a given measure of quality. PSO optimizes a problem by having a population of candidate solution of candidate solutions (particles), and moving these particle around in the search-space according to simple mathematical formulations over the particle's position and velocity.

Each particle's movement is influenced by its best known local position and is also guided toward the best known positions in the search-space, which are updated to better positions are found by other particles.

According to Fig.1, the basis of methods is as follows:

Each particle can be shown by its current speed and position, the most optimal position of each individual and the most optimal position of the surrounding. Having chosen the initial population Xi,Vi, the speed and position of each particle change around search space according to the Eq.(5.a-d):

$$X_i = [x_{1,i}\ x_{i,2} \cdots x_{n,i}]$$

$$V_i = [v_{1,i}\ v_{i,2} \cdots v_{n,i}]$$

$$V_{id}^{k+1} = V_{id}^{k} + \Gamma_1 \times r_1^k \times \left(V_{id}^{Lbest}\right) + \Gamma_2 \times r_2^k \times \left(V_{id}^{Gbest}\right) \qquad (5.a)$$

$$X_{id}^{k+1} = X_{id}^{k} + V_{id}^{k} \qquad (5.b)$$

$$V_{id}^{Lbest} = pbest_{id}^{k} - X_{id}^{k} \qquad (5.c)$$

$$V_{id}^{Gbest} = gbest_{id}^{k} - X_{id}^{k} \qquad (5.d)$$

Where In this equality, $V_{id}^{k}$and X_{id}^{k} separately stand for the speed of the particle “i” at its “k” times and the d-dimension quantity of its position; $pbest_{id}^{k}$ represents the d-dimension quantity of the individual “i” at its most optimist position at its “k” times.

$gbest_{id}^{k}$ is the d-dimension quantity of the swarm at its most optimal position. In order to prevent the particle from being far away from the searching space, the speed of the particle created at its each direction is confined between -vdmax, and vdmax. If the number of vdmax is too big, the solution is far beyond the best, otherwise the solution will be the local optimum; c1 and c2 represent the speeding figure.

Now, based on our aims in the paper and the PSO application, the defined objective function for this problem can be considered as: Fitness-Function≜ Number of polyhedral + Output Specification.

Where output Specification is determined as a summation of operational specifications such as settling time, over shoot, under shoot, steady state deviation and so forth.

IV. CFTOC DEFINITION FOR DC-DC CONVERTER

DC-DC power converters are used to convert an unregulated DC voltage input to a regulated DC voltage output with controllable magnitude (Stefan et al.2010). These converters are applicable to feed the DC motors and control their revolution and DC power supply.

The DC-DC converters have some topologies such as step-down, step-up, buck-boost, cuk and full bridge converter (Kassakian et al.1991). among which our method will be applied to Buck-Boost topology. The object is to obtain and stabilize the output around a regulated DC Voltage. The Buck-Boost converter is shown in Fig.2, and the block diagram of the object is shown in Fig.3.

Considering the states as x:=[vc il]T where vc is the capacitor voltage and il is the indicator current. In the first step, according to Fig.4.a-b, the converter can be modeled by PWA with continuous indicator current.

The assumed PWA system is as Eq. (6):

$$x_{k+1} = A_i x_k + B_i u_k + a_i \quad \begin{pmatrix} x_k \\ u_k \end{pmatrix} \in D_d$$

$$y_{k+1} = C_i x_k + D_i u_k + f_i \qquad (6)$$

Where $D := \cup_{i=1}^{N_D} D_d$ denotes a polyhedral partition of the domain D (state-input space) defined as $[x_{1,min}, x_{1,max}]\times[x_{2,min}, x_{2,max}]\times[0,1]$. And $N_D<\infty$ is the number of system dynamics.

When the switch is on, the system matrices are as follows:

$$A = \begin{bmatrix} \frac{-1}{X_C}\left(\frac{1}{R+R_C}\right) & 0 \\ 0 & \frac{-R_L}{X_L} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{X_L} \end{bmatrix}, C = \begin{bmatrix} \frac{R}{R+R_C} & 0 \end{bmatrix}, a = \begin{bmatrix} 0.05 \\ 0 \end{bmatrix}$$

And when the switch is off, the system matrices are as follows:

$$A = \begin{bmatrix} 0 & \frac{-1}{X_C}\left(\frac{1}{R+R_C}\right) \\ \frac{-1}{X_L}\left(\frac{1}{R+R_C}\right) & \frac{-1}{X_L}\left(\frac{RX_C}{R+R_C}\right) \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{X_L} \end{bmatrix}, C = \begin{bmatrix} \frac{R}{R+R_C} & \frac{R.R_C}{R+R_C} \end{bmatrix}, a = \begin{bmatrix} 0.001 \\ 0 \end{bmatrix}$$

The value of capacitor, capacitor internal resistor, inductor and inductor internal resistor are 200µF, 10mΩ, 1µH, 10mΩ respectively. The initial PID coefficients are K_P=1, K_I=0.001 and K_D=0.5. Input voltage is 15V and desired output voltage is 20V.

The optimal control of a Buck-Boost DC-DC converter with presented model is minimizing the performance index in Eq. (7)

$$J(U, x(0)) := \sum_{t=0}^{N-1} \|Qx(t)\|_1 + \|Ru(t)\|_1$$

This model is used in a MPC problem formulation, where performance is optimized under constrained on state and control input. The problem resolved by using MPC theorem, MPT (Kvasnica and Baotic2004), Choosing Prediction horizon T = 2 and weights in the cost function Q= [100;0 5], R=1. Resulting control law is PWA and continuous with 8 polyhedral regions (Fig.5).

As shown in Fig.6, output voltage has unfavorable undershoot and overshoot and a minor steady state error exists. Therefore the output voltage is not much desired.

As known, Complexity of CFTOC solution grows exponentially with increasing polyhedral partitions. Now, this result can be used as an input function of PSO method. In this step, PSO method is used to obtain desired system parameters not only to modify system performance but reduce the number of polyhedral of control law. In other words, an algorithm which is a combination of MPC-based solution of CFTOC problem with PSO method is going to be used which can reduce the number of polyhedral of PWA control law and improve the system performance simultaneously.

V. PSO APPLICATION TO SOLVE PROPOSED PROBLEM

According to previous sections, control variables of this problem are physical parameters of converter system and PID coefficients. The problem is solved as following steps:

1. Required data such as input voltage, output voltage, algorithm parameters like number of population and number of iteration are considered.
2. Initial population is created as following:

$$X_i = [R_{C_i}\ R_{L_i}\ X_{C_i}\ X_{L_i}\ K_{p_i}\ K_{I_i} K_{d_i}]$$

$$\text{initial population} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{N\ pop} \end{bmatrix}$$

3. The converter system specifications are measured based on initial parameters and then the MPC-based control law derived off-line by MPT is applied and the number of polyhedral regions is calculated. Finally objective function is defined as:

Fitness-Function= Number of polyhedral + output specifications,Where we consider output specifications as: Output specifications ≜ settling time + undershoot+ steady state deviation

4. The best solution among the total population is determined and population is updated based on (5.a-b).
5. For predefined iteration, steps 3, 4 are done iteratively.
6. The convergence condition is checked and the best solution is shown in output finally.

After implementation of these steps, the optimal system parameters are obtained in order to reach the defined purposes. Since system parameters are changed, may be the system application affected by this change and steady state error happened. Therefore the coefficients of PID controller calculated during the mentioned steps based on using PSO method and set (7) them in order to get minimum of steady error again.

The optimal system parameters, the number of polyhedral and new PID coefficients are given in the table (1).

The steady state error are vanished, settling time, overshoot and other output specification are very desirous (as shown in Fig.7.b). Although the number of new controller polyhedral partitions reduces from 8 to 2 (Fig.8).

VI. CONCLUSION

The solution of the optimal control of constrained DC-DC power system modeled by PWA is offered in different approaches. The MPC is one of the most significant algorithms to solve the aforementioned problem. Despite all the advantages of MPC strategy, the online computational complexity of solution depended on the number of polyhedral regions of explicit control law can be considered as a disadvantage. Some studies have focused on reducing the number of polyhedral; however, several objects such as stability and improvement of performance have not been paid much attention. In the paper, considering the problem as NP-hard, the problem based on using PSO and its main property to solve the NP-hard problems is approached. The aim of our study is to present a new algorithm which is a combination of MPC-based solution of CFTOC problem with PSO method that can reduce the number of polyhedral of PWA control law and improve the system performance simultaneously. According to the results, it is demonstrated that by using PSO to solve the considered problem, the number of polyhedral and the dependent complexity of CFTOC solution are reduced; the system performance such as percent of overshoot and settling time by achieving the optimal system parameters are desirable and the steady state error obtained by calculating the optimal PID coefficients reaches zero.

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