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CONCEPT DIAGRAM OF ALGEBRA LEARNING

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***Abstract*—Each cluster of data can easily describe features of knowledge structures. The purpose of this study is to analyze the concept diagram of Abstract Algebra concepts for university students and clustering based on concept proficiency. Besides, fuzzy clustering on concept proficiency expresses the cognitive characteristics. Methodology in this study is CAISM. it shows CAISM can provide useful information for cognition diagnosis. According to the results, some suggestions and recommendations for future investigation are discussed.**

***Index Terms*—Concept Structure, CAISM, Cognition Diagnosis.**

I. INTRODUCTION AND MOTIVATION

Empirical data is the paper-and-pencil assessment of Abstract Algebra course. This method can not only present the personal concept structure by hierarchical diagram, but also calculate the magnitude of mastery on each concept. Concept structure analysis (CSA) could provide individualized knowledge structure. CSA algorithm is the major methodology and it is based on fuzzy logic model of perception (FLMP) and interpretive structural modeling (ISM). CSA could display individualized knowledge structure and clearly represent hierarchies and linkage among concepts for each examinee.

Concept diagram is one method of assessment and it will improve remedial instruction. One is to analyze total score and the other is to analyze concept diagram. Total score of test is adopted by classical assessment to indicate achievement of students. However, this method has some shortcomings. For example, total score is hard to provide information of cognitive diagnosis and total score can not distinguish features of concepts involved in the items. Therefore, analysis of total score exist limitations. Moreover, task-takers of same total score with different response pattern have distinct concept diagram.

Teaching and learning is an interactive process between teachers and students. Many courses of abstract algebra are involved in teaching Abstract Algebra either formally in a college classroom or informally in an industrial setting. Regardless of the setting, a major concern of those who teach statistics in how to ensure that the student understand abstract algebra's ideas and are able to apply what they learn to real-world situations. Although teachers of abstract algebra often express frustration about difficulties students have learning and applying course material, many may be unaware of the growing body of research related to teaching and learning abstract algebra. It is needed to design a systematic way to realize student's understanding about their concept diagrams. However, each task-taker should have his own knowledge structure based on the viewpoint of cognition science.

Clustering base on concept proficiency in knowledge structures should be helpful for remedial instruction. Therefore, methodological development for both individualized concept structure and clustering is an important issue. In this study, foundations of clustering operation from fuzzy theory will be used. The empirical data is abstract algebra's test.

In this paper, Knowledge management of abstract algebra's Concepts was essential in educational environment. The purpose of this study is to provide an integrated method of fuzzy theory basis for individualized concept structure analysis. This method integrates Fuzzy Logic Model of Perception (FLMP) and Interpretive Structural Modeling (ISM). The combined algorithm CAISM (concept advanced interpretive structural modeling) can reveal individualized concept diagram [1] [2] [3], only brief discussion is considered in this study. Fuzzy clustering based on proficiency of concepts, which is derived from results of individualized concept structure, is adopted so that task-takers will be properly clustered. Students of the same total score with different response pattern will also be chose and discussed. Results of this research are prospective on the cognition diagnosis for courses of abstract algebra learning and help teachers design remedial instruction. However, CAISM could analyze individualized concepts structure based on the comparisons with concept structure of expert. It has great benefits if analysis of concept diagram is adopted. How to analyze concept diagram is a prospective methodology. It is because diagram could be easily understood and individualized concept diagram can improve realization for characteristics of concepts for each task-taker.

Cognition diagnosis can help represent knowledge structure [4] [5] [6]. It is a common viewpoint that human knowledge is stored in the form of structural relationship among concepts and their subordinate relationship is fuzzy, not crisp. There are some methodologies for concept structure analysis but little is known about methodologies of individualized concept structure [7] [8] [9] [10]. Therefore, the development for methodology of individualized concept structure is an important issue and it is essential for cognition diagnosis and pedagogy [11].

II. LITERATURE REVIEW

In this study, the integrated method of individualized concept structure based on fuzzy logic model of perception (FLMP) and interpretive structural modeling (ISM) will be developed. An example of empirical test data of abstract algebra's concept for students of learning deficiencies will also be analyzed and discussed. For the feasibility of remedial instruction based on the cognition diagnosis, clustering method is needed so that students within the same cluster own similar knowledge structures and students among different clusters have the most variance on knowledge structures [12] [13].

The algorithm of interpretive structural modeling is the foundation of this study and other topics will also discuss.

A. Interpretive Structural Modeling

The algorithm of interpretive structural modeling is the foundation of this study. They will be discussed as follows. Moreover, issue on the statistics assessment will be also discussed. The theory of interpretive structural modeling (ISM) is based upon discrete mathematics and graph theory. J. N. Warfield provided ISM and it aims to arrange elements in a hierarchical relation. For any set that contains elements, we

can make a hierarchical graph of all elements if the binary relationship among elements is known. Namely, the relationship of A_i and A_j must be acquired in advance.

The relationship could be expressed in the form of matrix $A=(a_{ij})_{K\times K}$. If $a_{ij}=1$ exists, A_i is the precondition of A_j . On the other hand, if it is $a_{ij}=0$, A_i is not the precondition of A_j . The analytical procedure of ISM is as follows.

The ISM adopts Boolean operation. The transitive closure is $\hat{A}=A\oplus A^2\oplus A^3\oplus\cdots A^P$ and $R=\hat{A}\oplus I=(A\oplus I)^P$ is reachability matrix. With transitive closure $\hat{A}$ and reachability matrix R, the hierarchical graph of elements in matrix $A=(a_{ij})_{K\times K}$ could be plotted. For example, let the $A=(a_{ij})_{K\times K}$ be as follows. Its relationship and hierarchies of elements is depicted in Fig. 1.

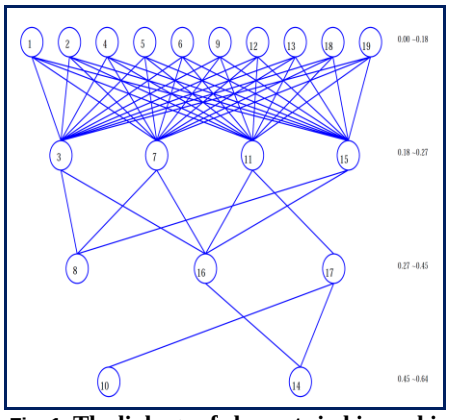

Fig. 1 The linkage of elements in hierarchies

B. Fuzzy Logic Model of Perception

Suppose there be a combination of two factors and there are levels within each factor. There are levels and levels for factor and respectively. The fuzzy true values are expressed. Fuzzy truth value and express the degree that the combination of and will support prototype [14] [15]. The probability that the combination of could be viewed as prototype can be expressed as follows [16]. The fuzzy true value is to express the degree that the combination of two levels from distinct factor will support prototype. The probability that this combination could be viewed as prototype is as follows

$$p(c_i\ ,o_j)=c_io_j[c_io_j+(1-c_i)(1-o_j)]^{-1} \quad (1)$$

C. Clustering and Fuzzy Theory

Cluster analysis is a data analysis tool to find proper clusters of a data set with the most homogeneity in the same cluster and the most heterogeneity among clusters. Statistics approaches include two categories of clustering, which are hierarchical and non-hierarchical. The k-means algorithm is a popular example of non-hierarchical clustering whereby the data is partitioned into -classes with the value of known a priori. Zadeh provides the fuzzy theory and it flourishes well and bring contributions to clustering analysis. Fuzzy clustering is based on the membership of sample points and fuzzy c-means is the most popular technique. In this study, Bezdek's algorithm is applied.

Suppose there be a data matrix with subjects and variables. The data matrix $X=(x_{nm})_{N\times M}$ is known. The membership matrix $U=(u_{cn})_{C\times N}$ and the group center matrix $V=(v_{cm})_{C\times M}$ are unknown with group number C . The objective function with optimization problem is applied and membership matrix and group center matrix are derived by way of iteration.

$$J(U,V)=\sum_{n=1}^{N}\sum_{c=1}^{C}(u_{cn})^q d^2(c,n) \quad (2)$$

$$u_{cn}=\frac{1}{\sum_{l=1}^{C}\left[\frac{d^2(c,n)}{d^2(l,n)}\right]^{\frac{1}{q-1}}} \quad \text{and} \quad v_{cm}=\frac{\sum_{n=1}^{N}(u_{cn})^q(x_{nm})}{\sum_{n=1}^{N}(u_{cn})^q} \quad (3)$$

Partition entropy $H(U;C)=\frac{-1}{N}\sum_{n=1}^{N}\sum_{c=1}^{C}u_{cn}\ln(u_{cn})$ and partition coefficient $F(U;C)=\frac{1}{N}\sum_{n=1}^{N}\sum_{c=1}^{C}(u_{cn})^q$ are used to decide the number of cluster. Larger partition coefficient and smaller partition entropy means the better partition number C will be used.

D. Integrated model and Procedure

The procedure of the integrated model is depicted in Figure 2. Firstly, FCM is to classify examinee based on their response pattern. Secondly, concept structure analysis (CSA) will analyze individualized knowledge structures. CSA includes three algorithms, which are AMC (algorithm for mastery of concept), ASC (algorithm for subordination of concepts) and AFISM (algorithm for fuzzy ISM). By the integrated procedure, Examinee within the same cluster represent similar knowledge structures and remedial instruction could be feasible based on the information of cognition diagnosis within the same cluster.

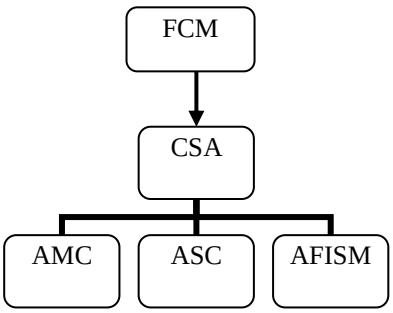

Fig. 2 Procedure of the Integrated Model

Three algorithms, AMC, ASC and AFISM, are combined in order to analyze individualized knowledge structure. Some basic definitions are as follows [1]. The integrated algorithms consist of three steps of algorithms, AMC, ASC and AFISM.

E. Abstract Algebra's Education and Assessment

In this study, the authors provide the empirical data for concepts of the course of abstract algebra from university students. To sum up, the methodology can improve knowledge management in classroom more feasible. It has great benefits if analysis of concept diagram is adopted. However, each task-taker should have his own knowledge structure based on the viewpoint of cognition science. Teaching and learning is an interactive process between teachers and students.

Students learn by constructing knowledge and active involvement in learning actives. It should not underestimate the difficulty students have in understanding basic concepts of

abstract algebra. It is often neglected how students understand basic concepts. Students will learn better if they receive consistent and helpful feedback on their performance. Therefore, it is important to evaluate what students know. Learning will be enhanced by investigating concept diagrams so that misconception and mastery of concepts will be understood. A better understanding of students' learning not only can be important references in determining success in abstract algebra, but also be beneficial to both teachers and students. Assessment will helps teachers understand the learning styles of students and adjust their own instructions accordingly. Furthermore, adaptively remedial instruction could depend on the information of students' shortcomings. In addition, students will also benefit by managing their learning process more efficiently. However, most researches related to abstract algebra learning and measurement adopt descriptive methods. Little is known about the analysis of knowledge structures. Therefore, the methodology to analyze concept diagrams about university students should be prospective.

In this paper, abstract algebra consists of three parts.

(A).Group Theory:

1.Understand the definitions and the properties of groups, and have the ability to distinguish different groups.
2.Understand the definitions and the properties of subgroups.
3.Understand the definitions and the properties of homomorphism and auto-morphism.

(B).Ring Theory:

1.Understand the definitions and the properties of rings.
2.Understand the definitions and the properties of integral domains, division rings, fields, ideals and quotient rings.
3.Understand the definitions and the properties of homomorphism and auto-morphism, and know some familiar ideals and rings.
4.Understand the definitions and the properties of polynomial rings.

(C).Field Extension and Galois Theory:

1.Understand the definitions and the properties of field extensions.
2.Know some familiar field extensions.
3. Understand the significance of the fundamental theorem of Galois theory.

F. METHODOLOGY AND ALGORITHM

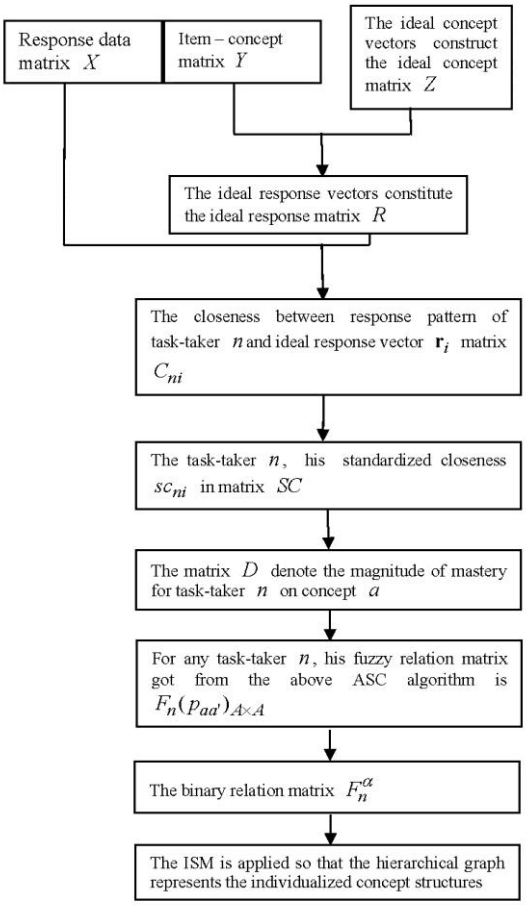

Fig. 3 The process of CAISM algorithm

In this study, CAISM is developed to integrate method of fuzzy logic model of perception (FLMP) and interpretive structural modeling (ISM). The method analyzes concept diagram based on response data matrix and item attribute matrix. It will reveal the individualized concept diagram and relationship. The process of CAISM algorithm is depicted in Fig. 3

III. EMPIRICAL ANALYSIS

The course of abstract algebra assessment is designed by the authors. There are 251 university students and take elementary linear algebra and advance of abstract algebra courses. There are 10 concept attributes within each item and they are depicted in Table1.

TABLE I. CONCEPT ATTRIBUTES OF TEST

Concepts	Concept Attributes
1	Groups and distinguish different groups
2	The properties of subgroups
3	Homomorphism of group
4	The Properties of rings
5	Division rings, and quotient rings
6	Homomorphism of ring
7	The properties of polynomial rings.
8	Properties of field extensions.
9	Know some familiar field extensions.
10	Fundamental theorem of Galois theory

The test includes 25 items. The data set comes from the National Taichung University of Education used in the

empirical study with learning Abstract Algebra. The item concept matrix is depicted in Table 2. For example, from item 1 to item 7, these items measure concept 1. From item 3 to item 7, these items measure concept 2. From item 5 to item 7, these items measure concept 3. Similarly, From concept 1 to concept 10, they are random sampling, sampling distribution, sampling distribution of mean, central limit theorem and sampling distribution of ratio. All these items are dichotomous and is decided in the CAISM algorithm.

TABLE II. ITEM CONCEPT MATRIX

Items	Concept					Items	Concept				
	1	2	3	4	5		6	7	8	9	10
A1	1	0	0	0	0	B7	1	0	0	0	0
A2	1	0	0	0	0	B8	1	0	0	0	0
A3	1	1	0	0	0	B9	1	0	0	0	0
A4	1	1	0	0	0	B10	0	1	0	0	0
A5	1	1	1	0	0	B11	0	1	1	0	0
A6	1	1	1	0	0	C1	0	1	1	1	0
A7	1	1	1	0	0	C2	0	0	1	0	0
B1	0	0	0	1	0	C3	0	0	1	0	0
B2	0	0	0	0	1	C4	0	1	1	0	1
B3	0	0	0	0	1	C5	0	0	1	0	1
B4	0	0	0	1	1	C6	0	1	1	1	1
B5	0	0	0	1	1	C7	0	1	1	1	1
B6	0	0	0	1	1						

The data of task-takers ID 106 is depicted in Table 3. In Fig. 4, the symbol of each concept and magnitude of mastery on concept of task-taker ID 106 is shown. In task-taker ID 106, the mastery of all concepts except concept 10 is 1.00 and these are equivalent relation. These concepts are the precondition of concept 10. From the item concept matrix of table 2, we can Obtain verification.

TABLE III. RESPONSE PATTERN AND TOTAL SCORE OF ID 106

ID	Response Pattern	Total Score
106	111111111111111111110000	21

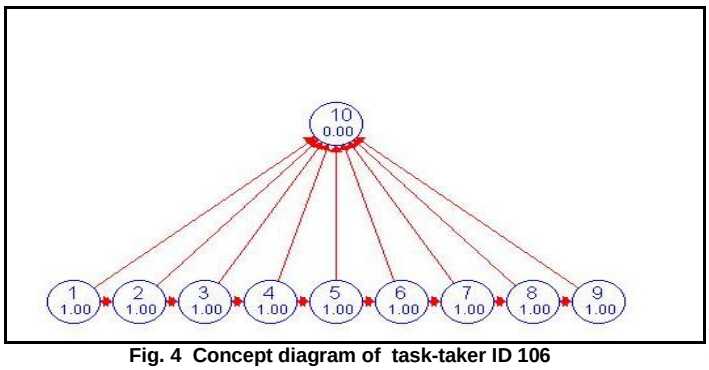

Fig. 4 Concept diagram of task-taker ID 106

In task-taker ID 76, the mastery of all concepts except concept 1, concept 2 and concept 3 is 1.00 and these are equivalent relation. These concepts are the precondition of concept 1, concept 2 and concept 3. The mastery of concept and concept 2 is 0.50 and these are equivalent relation. These concepts are the precondition of concept 1.

TABLE IV. RESPONSE PATTERN AND TOTAL SCORE OF ID 76

ID	Response Pattern	Total Score
76	000011111111111111111111	21

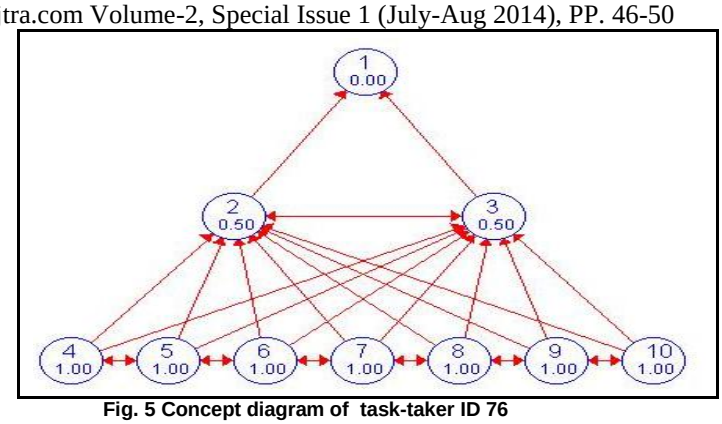

Fig. 5 Concept diagram of task-taker ID 76

Compare to task-taker ID 76 and task-taker ID 106, they get same total score, but Concept diagram of task-taker ID 76 and task-taker ID 106 is different, It implies that difficulties of these concepts are quite different. Although CAISM could provide the concept diagram of each task-taker.

To sum up, this integrated algorithm could improve the assessment methodology of cognition diagnosis and manage the knowledge structures of abstract algebra's concepts easily.

In task-taker ID 9, the mastery of concept 8 is .51. It is the basic concept of all concepts. For task-taker ID 9, The mastery of concept 7 is 0.46. The precondition of concept 7 is concept 1, concept 2, concept 3, concept 4, concept 6, concept 9 and concept 10 and Only concept 1, concept 2 and concept 3 are equivalent relation. The concept 5 is isolated concept.

TABLE V. RESPONSE PATTERN AND TOTAL SCORE OF ID 9

ID	Response Pattern	Total Score
9	0110110101100001000110100	11

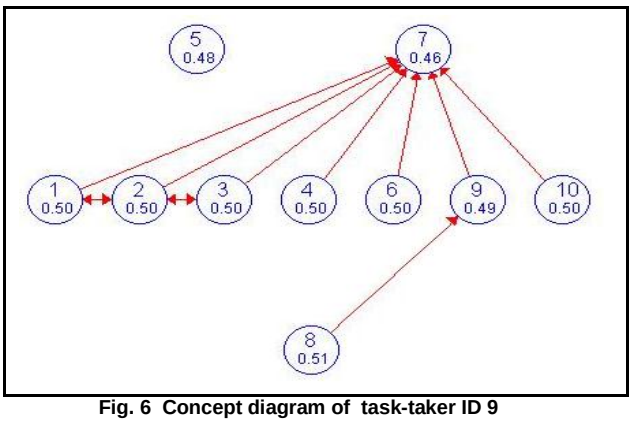

Fig. 6 Concept diagram of task-taker ID 9

In task-taker ID 205, the mastery of concept 1 is .52. It is the basic concept of all concepts. For task-taker ID 205, The mastery of concept 8 is 0.45. The precondition of concept 8 is concept 2, concept 5, concept 6 and concept 9. None concepts is equivalent relation. The concept 4 is isolated concept.

TABLE VI. RESPONSE PATTERN AND TOTAL SCORE OF ID 205

ID	Response Pattern	Total Score
205	10111111101100000010100 10	13

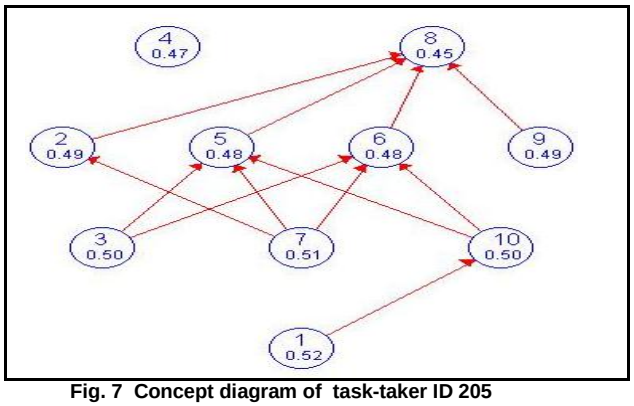

Fig. 7 Concept diagram of task-taker ID 205

Compare to task-taker ID 9 and task-taker ID 205, Concept diagram of task-taker ID 9 and task-taker ID 205 is different, Knowledge Management of Mathematics Concepts was essential in educational environment. The purpose of this study is to provide an integrated method of fuzzy theory basis for individualized concept structure analysis. In the task-taker ID 9 of Fig. 6, concept 8 is more easily than others. But In the task-taker ID 205 of Fig. 7, concept 8 is more difficultly than others.

In addition, empirical test data of abstract algebra for university students are discussed. It shows that knowledge structures will be feasible for remedial instruction [17].

This procedure will also useful for cognition diagnosis. To sum up, this integrated algorithm could improve the assessment methodology of cognition diagnosis and manage the knowledge structures of Abstract Algebra Concepts easily.

An example of empirical test data of linear algebra concept for students of learning deficiencies will also be analyzed and discussed. For the feasibility of remedial instruction based on the cognition diagnosis, clustering method is needed so that students within the same cluster own similar knowledge structures and students among different clusters have the most variance on knowledge structures [18].

IV. Conclusions

However, each task-taker should have own knowledge structure based on the viewpoint of cognition science. In this paper, each cluster of data can easily describe features of knowledge structures. We can manage the knowledge structures of Abstract Algebra Concepts to construct the model of features in the pattern recognition completely. An integrated method of FLMP and ISM for analyzing individualized concept structure is provided. With this integrated algorithm, the graphs of concept structures will display the characteristics of knowledge structure. This result corresponds with foundation of cognition diagnosis in psychometrics [11]. The correct ration of concepts varies. It implies that difficulties of these concepts are quite different. Although CAISM could provide the concept diagram of each task-taker, it is unfeasible to display the concept diagrams of all task-takers respectively.

As shown in Table 3 and Table 4, two task-takers , ID 106 and ID 76, have the same total score 21 with different response pattern. As shown in Fig. 4 and Fig. 5, these two task-takers have varied concept diagrams. In addition to the distinction on mastery of concepts, the linkages and prerequisite relationship among concepts also differ.

This phenomenon supports viewpoints of cognitive psychology and psychometrics that response pattern could distinguish characteristics of concept diagram, but not total score. An integrated method of individualized concept structure and fuzzy clustering on proficiency of concepts could correspond with issue of cognition diagnosis in psychometrics. This study investigates the concept diagrams of university students. One is concluded that the study help understand personal concepts diagram. Thus, teachers could design the adaptively remedial instruction. Another finding is that concept diagram reveals personal misconceptions or erroneous linkage among concepts. Future research could apply CAISM to another fields or subjects. Furthermore, it is prospective to integrate CAISM with instructional technology so that e-learning approach of remedial instruction becomes feasible.

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