International Journal of Technical Research and Applications e-ISSN: 2320-8163,
www.ijtra.com Special Issue 32 (September, 2015), PP. 20-24

A COMPARISON OF S&P 500 INDEX FORECASTING MODELS OF ARIMA, ARIMA WITH GARCH-M AND ARIMA WITH E-GARCH

Saowanit Sukparungsee
Department of Applied Statistics
Faculty of Applied Science
King Mongkut's University of Technology North Bangkok
Bangkok, Thailand
swns@kmutnb.ac.th

Abstract— **This objective of this research is to compare the forecasting models of S&P 500 index with 3 models- ARIMA, ARIMA with GARCH and ARIMA with E-GARCH. The secondary data are used to predict daily the values of S&P 500 since January, 1 to October, 31 2014. The performance of forecasting models in term of accuracy is measured by using of Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE). The most appropriate model of S&P 500 is ARIMA with E-GARCH which given the minimal MAPE and RMSE.**

Index Terms— **S&P 500, ARIMA with E-GARCH, MAPE, RMSE**

I. INTRODUCTION

To start and expand the business to the Company are issued to them. Funding is important to meet the needs of prospective customers to get more done. Investments would be something even more important. To be able to accommodate a large number of funding sources, the use of shareholder capital. Loan from the financial institutions can help the business expand. It is a channel for raising funds through a publicly traded company. Currently raising funds from shareholders through stock exchange mechanism is essentially the original owner must be visible and allow other parties to join a business owner with fundraising ideas are incurred due to the previous owners did [7]. So it had to take the company to distribute its shares to the public and investors to buy shares of the company and share ownership, the proportion of shares held by him in 1698, or more than 300 years ago found evidence of "shares" (Stock) and "futures" (Commodity) turns in a cafe in London, England, in 1790 (or 215 years ago). The US Government has released the first public sale of bonds and stocks. Born officially in two years later, in the early 17th century, some believed to be the world's first stock market began the first in the Netherlands. Nowadays, the market place in countries around the world, including Europe and Asia, the stock market in each country at least 30 percent of the North American continent have more than 20 towns.

The S & P 500 is an index derived from the Standard and Poor's 500 conducted by the Institute named Standard and Poor's Corporation is an index derived from the company listed in the United States, over 500 companies, which have applied in the selection process. As the liquidity of the shares of industry, etc. using the market value of each company to calculate the S & P 500 is an index measuring overall market conditions, and is often used to refer to the US economy.

Therefore, the factors studied Forecasting model is appropriate and effective for the index S & P 500 by the models studied in this research is the ARIMA, ARIMA with GARCH and ARIMA with E-GARCH models will have a model that is appropriate, particularly useful for planning and investment decisions in the future [1, 2, 4, 6].

II. RESEARCH METHODOLOGY

Secondary data and theories used in research.

1. Selection of Data S&P average share price information daily since January 1, 2014 to October 31, 2014, divided into two series is set for the creation of the model. There are also testing the accuracy of forecasting.
2. Testing stability data (or Stationary), and the unit root test (Unit Root Test) to check on the rest of the data to determine the predictive models. The random (ε_t) is called stationary, with the following format.

The data come from random processes (Random Process) must put it to the test that are unstable or not by testing the unit root (Unit Root) equation below.

$$X_t = \rho X_{t-1} + \varepsilon_t \tag{1}$$

where ρ is the correlation of the population.

The hypothesis testing is as following:

$$H_0 : \rho = 1 \text{ vs } H_1 : |\rho| < 1.$$

The model for testing Dickey-Fuller Test (DF) which was presented by Dickey and Fuller in 1981 [3] to make it easier to test unit root (Unit Root) of the time series that are stationary.

Then, differ X_{t-1} both sides of the equation (1), as will be shown below

$$\Delta X_t = \theta X_{t-1} + \varepsilon_t \qquad (2)$$

The hypothesis testing is as following:

$H_0 : \theta = 0$ vs $H_1 : \theta \neq 0.$

This test can be rewritten into the equation is the following:

i. no constant and time trend:

$$\Delta X_t = \theta X_{t-1} + \varepsilon_t$$

ii. only constant:

$$\Delta X_t = \alpha + \theta X_{t-1} + \varepsilon_t$$

iii. both constant and time trend:

$$\Delta X_t = \alpha + \beta_t + \theta X_{t-1} + \varepsilon_t$$

where θ is the parameters of the unit root tests and β_t is a constant trend.

Later in the year 1984 [6], Said and Dickey have proposed Augmented Dickey-Fuller Test (ADF) to increase the number of lagged difference terms in the equation to solve autoregressive (serial correlation) the following equation:

i. no constant and time trend:

$$\Delta X_t = \theta X_{t-1} + \sum_{i=1}^{\rho} \phi_1 \Delta X_{t-1} + \varepsilon_t$$

ii. only constant:

$$\Delta X_t = \alpha + \theta X_{t-1} + \sum_{i=1}^{\rho} \phi_1 \Delta X_{t-1} + \varepsilon_t$$

iii. both constant and time trend:

$$\Delta X_t = \alpha + \beta_t \theta X_{t-1} + \sum_{i=1}^{\rho} \phi_1 \Delta X_{t-1} + \varepsilon_t$$

To verify that the data are stationary or moving them to compare the t statistic was calculated with the crisis (Critical Value) in the table ADF using statistical t (t-statistic) which has the formula below.

$$t = \frac{\hat{\theta}}{S.E._{\hat{\theta}}}.$$

3. Analysis Forecasting 3 Models

3.1 Using predictive models with ARIMA as following equation

$$\left(1 - \phi_1 B - \ldots - \phi_p B^p\right) X_t = \phi_0 + (1 - \theta_1 B - \ldots - \theta_q B^q)\varepsilon_t \qquad (3)$$

3.2 Using predictive models with ARIMA - GARCH as following equation

$$\begin{aligned} &(1-B)^d \left(1 - \phi_1 B - \ldots - \phi_p B^p\right) X_t \\ &= \phi_0 + (1 - \theta_1 B - \ldots - \theta_q B^q)\varepsilon_t + \delta_1 \sigma_t^{1/2} \end{aligned} \qquad (4)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2$$

3.3 Using predictive models with ARIMA - EGARCH as following equation

$$\begin{aligned} &(1-B)^d \left(1 - \phi_1 B - \ldots - \phi_p B^p\right) X_t \\ &= \phi_0 + (1 - \theta_1 B - \ldots - \theta_q B^q)\varepsilon_t \\ &\log\left(\sigma_t^2\right) = \omega + \alpha\left(\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - \sqrt{\frac{2}{\pi}}\right) + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta \log(\sigma_{t-1}^2) \end{aligned} \qquad (5)$$

4. Forecasting Measurement Criteria

Considering the Mean Absolute Percentage Error (MAPE) and the Root Mean Square Error (RMSE) from the forecasting models based on ARIMA, ARIMA–GARCH and ARIMA-EGARCH models. By comparison, the forecast error of the S&P 500 index, the formula of those errors is calculated as follows.

4.1 Mean Absolute Percentage Error: MAPE

$$MAPE = \frac{1}{n}\sum_{i=1}^{n} \frac{|y_t - \hat{y}_t|}{y_t}(100).$$

4.2 Root Mean Square Error: RMSE

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_t - \hat{y}_t)^2}.$$

If the MAPE and RMSE of forecasting models which give a minimum value of error that model predict accurately and optimally.

III. The Results

In this study, the models predict stock price index S & P 500 3 models, each model with the following steps.

3.1 Analysis of ARIMA model

3.1.1 Test Unit Root Test

Test Unit Root Test of time series data was to see if the time series data is stationary or unstable. Non-stationary (I (d); d> 0; Integration of order 0) to avoid data are average (Mean) and volatility (Variances) is not constant over time different. The tested using Augmented Dickey - Fuller Test (ADF) to start the test data at the Level or the Order of Integration is 0 or 1 in order to compare the statistics ADF with MacKinnon Critical level at 1% and 5%. 10% if the ADF statistics show that over the MacKinnon Critical time series that looks unstable. (Non-stationary), which was edited by finding the difference or until the next time series data, it looks stationary.

TABLE I. TEST UNIT ROOT

At Level							
		None		Intercept		Trend and Intercept At	
S&P 500	lag	ADF statistic	% critical value	ADF statistic	% critical value	ADF statistic	% critical value
In dex	0	1.94	1 -2.58	-1.14	1-3.46	-3.14	1-3.46
			5 -1.94		5 -2.88		5-3.43
			10-1.62		10-2.57		10 - 3.14
At 1 Different							
		None		Intercept		Trend and Intercept At	
S&P 500	lag	ADF statistic	% critical value	ADF statistic	% critical value	ADF statistic	% critical value
D index	0	- 11.47	1 -2.58	- 11.44	1 -3.46	- 11.41	1-4.01
			5 -1.94		5 -2.88		5 -3.43
			10-1.62		10-2.57		10 - 3.14

Test results for S & P 500 stock compare the statistics from the ADF with the MacKinnon Critical level at 1%, 5% and 10% of the time series data. It appears that there is nonstationary time-series data. The statistics show that the ADF is greater than MacKinnon Critical information is not steady. Non-stationary will be unpredictable that it needs to find the difference between No. 1 (1 Difference) in order to compare the statistics with the ADF MacKinnon Critical level at 1%, 5% and 10% of the time series data. Statistics show that the ADF is less than the MacKinnon Critical, so that it looks stationary time series data (Stationary).

3.1.2 Identification

For the format of the model ARIMA, you must consider the correlogram of time series data at different sequence 1 (1 Difference) stock price S & P500 that are stationary and can find the form of the model. By defining the model to find Autoregressive AR (p) and Moving Average MA (q), which is determined by the value of Autocorrelation Function (ACF) and the Partial Autocorrelation (PAC) to create a model ARIMA (p, d, q) considers that the ACF and PAC exceeded outside the confidence interval, 95% to be a model. The SARIMA5 (0, 1, 1) is the best fit. As you can see in Table II of the Autocorrelation Function (ACF) has a value that exceeded outside the confidence interval, only one value and in respect of the Partial Autocorrelation (PAC) is beyond out of range. Confidence is only one value as well.

3.1.3 Parameter Estimation

From Table III, the coefficient of MA (5) is equal to -0.958371, which t-statistic is different from zero significance level of 5%, with the Akaike Information Criterion (AIC) value is equal to -7.014651, Schwarz. Information Criterion (SIC) is equal to 2.186162, the Durbin-Watson statistic is equal to -6.998765, the Residual sum of square is equal to 0.010997 and the R-squared is 0.521482, which means that the parameters of the model can explain the dependent variable was 52.1483%.

TABLE II. CORRELOGRAM

<table>
<tr><td colspan="7">Included observations: 211</td></tr>
<tr><td>Auto correlation</td><td>Partial Correlation</td><td></td><td>AC</td><td>PAC</td><td>Q-Stat</td><td>Prob</td></tr>
<tr><td>*|. |</td><td>*|. |</td><td>1</td><td>-0.13</td><td>-0.13</td><td>53.37</td><td>0.00</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>2</td><td>-0.03</td><td>-0.05</td><td>53.55</td><td>0.00</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>3</td><td>-0.03</td><td>-0.04</td><td>53.80</td><td>0.00</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>4</td><td>0.09</td><td>0.08</td><td>55.43</td><td>0.00</td></tr>
<tr><td>****|. |</td><td>****|. |</td><td>5</td><td>-0.53</td><td>-0.52</td><td>66.44</td><td>0.00</td></tr>
<tr><td>.|* |</td><td>.|. |</td><td>6</td><td>0.11</td><td>0.02</td><td>69.15</td><td>0.00</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>7</td><td>0.06</td><td>0.04</td><td>69.99</td><td>0.00</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>8</td><td>0.13</td><td>0.13</td><td>73.48</td><td>0.00</td></tr>
<tr><td>*|. |</td><td>.|. |</td><td>9</td><td>-0.06</td><td>0.03</td><td>74.25</td><td>0.00</td></tr>
<tr><td>.|. |</td><td>**|. |</td><td>10</td><td>0.06</td><td>-0.29</td><td>75.14</td><td>0.00</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>11</td><td>-0.05</td><td>-0.01</td><td>75.73</td><td>0.00</td></tr>
<tr><td>*|. |</td><td>.|. |</td><td>12</td><td>-0.06</td><td>-0.03</td><td>76.62</td><td>0.00</td></tr>
<tr><td>*|. |</td><td>.|. |</td><td>13</td><td>-0.13</td><td>-0.02</td><td>80.48</td><td>0.00</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>14</td><td>0.01</td><td>-0.05</td><td>80.51</td><td>0.00</td></tr>
<tr><td>.|. |</td><td>**|. |</td><td>15</td><td>-0.04</td><td>-0.27</td><td>80.94</td><td>0.00</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>16</td><td>0.05</td><td>0.02</td><td>81.48</td><td>0.00</td></tr>
<tr><td>.|*******|</td><td>.|*******|</td><td>1</td><td>0.974</td><td>0.974</td><td>208.60</td><td>0.00</td></tr>
<tr><td>.|*******|</td><td>.|. |</td><td>2</td><td>0.951</td><td>0.057</td><td>408.55</td><td>0.00</td></tr>
<tr><td>.|*******|</td><td>.|. |</td><td>3</td><td>0.927</td><td>-0.039</td><td>599.29</td><td>0.00</td></tr>
<tr><td>.|*******|</td><td>.|. |</td><td>4</td><td>0.903</td><td>-0.012</td><td>781.10</td><td>0.00</td></tr>
<tr><td>.|*******|</td><td>.|. |</td><td>5</td><td>0.877</td><td>-0.044</td><td>953.50</td><td>0.00</td></tr>
<tr><td>.|*******|</td><td>.|. |</td><td>6</td><td>0.854</td><td>0.031</td><td>1117.7</td><td>0.00</td></tr>
<tr><td>.|****** |</td><td>.|. |</td><td>7</td><td>0.830</td><td>-0.010</td><td>1273.7</td><td>0.00</td></tr>
<tr><td>.|****** |</td><td>.|. |</td><td>8</td><td>0.805</td><td>-0.045</td><td>1421.1</td><td>0.00</td></tr>
<tr><td>.|****** |</td><td>*|. |</td><td>9</td><td>0.777</td><td>-0.081</td><td>1559.1</td><td>0.00</td></tr>
<tr><td>.|****** |</td><td>.|. |</td><td>10</td><td>0.752</td><td>0.031</td><td>1688.7</td><td>0.00</td></tr>
<tr><td>.|****** |</td><td>.|. |</td><td>11</td><td>0.727</td><td>0.005</td><td>1810.5</td><td>0.00</td></tr>
<tr><td>.|***** |</td><td>.|. |</td><td>12</td><td>0.703</td><td>0.012</td><td>1925.0</td><td>0.00</td></tr>
<tr><td>.|***** |</td><td>.|. |</td><td>13</td><td>0.681</td><td>0.027</td><td>2033.1</td><td>0.00</td></tr>
<tr><td>.|***** |</td><td>.|. |</td><td>14</td><td>0.660</td><td>0.002</td><td>2135.2</td><td>0.00</td></tr>
<tr><td>.|***** |</td><td>.|. |</td><td>15</td><td>0.642</td><td>0.046</td><td>2232.2</td><td>0.00</td></tr>
<tr><td>.|***** |</td><td>.|. |</td><td>16</td><td>0.625</td><td>0.018</td><td>2324.6</td><td>0.00</td></tr>
</table>

TABLE III. COEFFICIENT AND STATISTICAL MODEL SARMA(5) (0,1,1)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(5)	-0.958	0.012	-80.565	0.000
R-squared	0.521	Durbin-Watson stat		2.186
Sum squared resid	0.011	Akaike info criterion		-7.014
Schwarz criterion		-6.999		

The model of SARIMA5 (0,1,1) is following

$$\log\left(index\right) = -0.958371\varepsilon_{t-1} + \varepsilon_t .$$

3.1.4 Diagnostics Checking

In the process of diagnostics checking, the Q-statistic to test the properties of White noise of error estimated that Q-statistic model of SARIMA5 (0,1,1). A Probability values greater than 0.05 that indicates the error estimates of the model is a White noise or the error has a normal distribution with mean is zero and variance is constant. This means that the model is validated for accuracy and that are suitable for use in forecasting the future.

3.2 Analysis of ARIMA with GARCH-M

3.2.1 Parameter Estimation

Based on the model SARIMA5 also found that the model is not the most appropriate model. Then, adding to the GARCH-M model to a model that is more accuracy.

TABLE IV. COEFFICIENT AND STATISTICAL MODEL SARMA(5) (0,1,1) WITH GARCH-M(1,1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	-0.001	1.68E-05	11.922	0.00
MA(5)	-0.962	0.007817	-123.116	0.00
VARIANCE EQUATION				
C	9.23E-06	2.34E-06	7.653	0.00
RESID(-1)^2	0.203	0.093	2.172	0.03
GARCH(-1)	0.637	0.170	3.739	0.00
R-squared	0.521	Durbin-Watson stat		2.19
S.E. of regression	0.007	Akaike info criterion		-7.05
Sum squared resid	0.011	Schwarz criterion		-6.98

The model of SARIMA5 (0,1,1) with GARCH-M (1,1) is following

$$\log\left(index\right) = -0.962424\varepsilon_{t-1} - 0.001093\sigma_t^{\frac{1}{2}}\varepsilon_t$$

$$\sigma_t^2 = 0.000923 + 0.202502\varepsilon_{t-1}^2 + 0.636619\sigma_{t-1}^2$$

From Table 4.6, coefficient of MA (5) is equal to -0.962424, the coefficient of Variance Equation include RESID(-1)^2 and GARCH (-1) equal to 0.203 and 0.637, respectively, the value of Z-statistic significantly different from zero at level of 1% with an Akaike Information Criterion (AIC) is equal to -7.05, Schwarz Information Criterion (SIC) is equal to 2.19076, the Durbin-Watson statistic is equal to -6.976, the Residual sum of square and the R-squared are equal to 0.011 and 0.52, respectively. This means that the parameters of the model can explain the dependent variable was 52.1078%.

3.2.2 Diagnostic Checking

In the process of diagnostic checking, the Q-statistic to test the properties of White noise of error estimated that Q-statistic model of SARIMA5 (0,1,1) with GARCH-M (1,1) p-value is greater than 0.05. All values are not different from zero significance level of 0.05 indicating that the discrepancies at an estimate of the model is a White noise or the error has a normal distribution. This means that the model is validated for accuracy and that are suitable for use in forecasting the future.

3.3 Analysis of ARIMA with E-GARCH

3.3.1 Parameter Estimation

The model of SARIMA5 (0,1,1) with E-GARCH is following

$$\log\left(index\right) = -0.962663\varepsilon_{t-1} + \varepsilon_t$$

$$\log(\sigma_t^2) = -1.132532 + 0.016759\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right|$$

$$-0.257068\frac{\varepsilon_{t-1}}{\varepsilon_{t-1}} + 0.88757310g(\sigma_{t-1}^2).$$

TABLE V. COEFFICIENT AND STATISTICAL MODEL SARMA(5) (0,1,1) WITH E-GARCH

Variable	Coefficient	Std. Error	z-Statistic	Prob.
MA(5)	-0.963	0.008	-125.004	0.000
Variance Equation				
C(2)	-1.133	0.313	-3.613	0.000
C(3)	0.0168	0.006	16.427	0.000
C(4)	-0.257	0.066	-3.869	0.000
C(5)	0.888	0.030	29.878	0.000
R-squared	0.521	Akaike info criterion		-7.127
Sum squared res	0.011	Schwarz criterion		-7.04
Durbin-Watson stat		2.191		

From Table IV., the coefficient of MA (5) is equal to -0.962663, the coefficient of Variance such C(2), C(3), C(4), C(5) are equal to -1.132532, 0.016759, 0.887573 and -0.257068, respectively, which are different from the Z-statistic significant level of 1% with an Akaike Information Criterion (AIC) is equal to the value -7.12657, Schwarz Information Criterion (SIC) of -7.04714. The Durbin-Watson statistic is equal to 2.190836, the total mobile and squared correlation coefficient are equal to 0.011009 and 0.520935, respectively. It means that the parameters of the model can explain the dependent variable was 52.0935%.

4.3.2 Diagnostic Checking

In the process of diagnostic checking, the Q-statistic to test the properties of White noise of error estimated that Q-statistic model of SARIMA5 (0,1,1) with E-GARCH. The p-value is greater than 0.05. All values are not different from zero significance level of 0.05 indicating that the discrepancy at an estimate of the model is a White noise or the error has a normal distribution. This means that the model is validated for accuracy and that are suitable for use in forecasting the future.

IV. SUMMARY AND CONCLUSION

The study forecast the S&P 500 index, using the technique to study three models also include forecasting models SARIMA 5, SARIMA5 with GARCH-M and E-GARCH model, compare the accuracy of the forecast. The results appear Table VI

TABLE VI. MAPE AND RMSE OF S&P 500 WITH ARIMA, ARIMA WITH GARCH-M AND ARIMA WITH E-GARCH

	ARIMA	GARCH-M	E-GRACH
MAPE	0.423	0.681	**0.422**
RMSE	9.738	14.353	**9.653**

Table VI considering the MAPE and RMSE of the best model concluded that the model can forecast the S&P 500 index is the model of SARIMA5 (0,1,1) with E-GARCH (1,1). This model obtained the minimal of MAPE = 0.422 and RMSE=9.653.

V. ACKNOWLEDGMENT

The author would like to express my gratitude to Faculty of Sciences, King Mongkut's University of Technology, North Bangkok, Thailand for supporting research grant.

REFERENCES

[1] J.S. Armstrong, *Evaluating forecasting methods, in J.S. Armstrong (ed.)*, Principles of Forecasting. Norwell, MA: Kluwer Academic Publishers, 2001.

[2] G.E.P. Box and G. Jenkins, *Time Series Analysis, Forecasting and Control*, Holden-Day, San Francisco, 1970.

[3] R.L. Hyndman, M. Akram and B.C. Archibald, "The admissible parameter space for exponential smoothing models," *Annals of the Institute of Statistical Mathematics,* Vol. 60, pp. 407-426, 2008.

[4] G. Li, H. Song, and S.F. Witt, "Recent developments in econometric modeling and Forecasting," Journal of Travel Research, Vol.44, pp. 82-99, 2005.

[5] E. Lawrence, K. Heitmann, M. White, D. Higdon, C. Wagner, S. Habib, and B. Williams, The Coyote Universe III: Simulation Suite and Precision Emulator for the Nonlinear Matter Power Spectrum, The Astrophysical Journal, Vol. 713, pp. 1322–1331, 2010.

[6] S. Makridakis, A. Andersen, R. Carbone, R. Fildes, M. Hibon, R. Lewandowski, J. Newton, E. Parzen, and R. Winkler, *The Forecasting Accuracy of Major Time Series Methods*, Chichester, England, John Wiley, 1984.

[7] N. Terui, and V. Dijk, "Predictability in the shape of the term structure of interest rates," International Journal of Forecasting, Vol.18, pp. 421–443, Hong Kong, 2002.