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SPLIT BLOCK SUBDIVISION DOMINATION IN GRAPHS

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Abstract: A dominating set $D \subseteq V[SB(G)]$ is a split dominating set in $[SB(G)]$. If the induced subgraph $\langle V[SB(G)] - D\rangle$ is disconnected in $[SB(G)]$. The split domination number of $[SB(G)]$ is denoted by $\gamma_{ssb}(G)$, is the minimum cardinality of a split dominating set in $[SB(G)]$. In this paper, some results on $\gamma_{ssb}(G)$ were obtained in terms of vertices, blocks, and other different parameters of G but not members of $[SB(G)]$. Further, we develop its relationship with other different domination parameters of G.

Key words: Block graph, Subdivision block graph, split domination number.

[I] INTRODUCTION

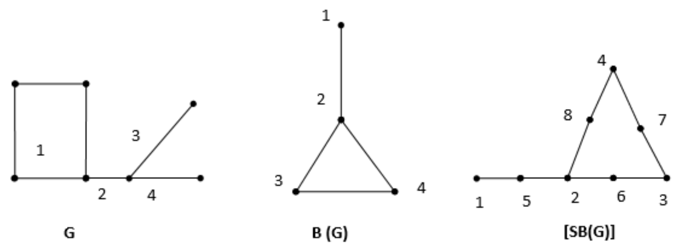
All graphs considered here are simple, finite, nontrivial, undirected and connected. As usual p, q *and* n denote the number of vertices, edges and blocks of a graph G respectively. In this paper, for any undefined term or notation can be found in F. Harary [3] and G .Chartrand and PingZhang [2]. The study of domination in graphs was begin by O.Ore [5] and C.Berge [1].

As usual, The minimum degree and maximum degree of a graph G are denoted by $\delta(G)$ *and* $\Delta(G)$ respectively. A vertex cover of a graph G is a set of vertices that covers all the edges of G.The vertex covering number $\alpha_0(G)$ is a minimum cardinality of a vertex cover in G.The vertex independence number $\beta_0(G)$ is the maximum cardinality of an independent set of vertices. A edge cover of G is a set of edges that covers all the vertices. The edge covering number $\alpha_1(G)$ of G is minimum cardinality of a edge cover. The edge independence number $\beta_1(G)$ of a graph G is the minimum cardinality of an independent set of edges.

A set of vertices $D \subseteq V(G)$ is a dominating set. If every vertex in $V - D$ is adjacent to some vertex in D.The Domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set in$G$.

A dominating set D of a graph G is a split dominating set if the induced subgraph $\langle V - D\rangle$ is disconnected. The split domination number $\gamma_s(G)$ of a graph G is the minimum cardinality of a split dominating set .This concept was introduced by $Kulli$[4]. A dominating set D of G is a cototal dominating set if the induced subgraph $\langle V - D\rangle$ has no isolated vertices. The cototal domination number $\gamma_{cot}(G)$ of G is the minimum cardinality of a cototal dominating set. See [4]

The following figure illustrate the formation of $[SB(G)]$ of a graph G



The domination of split subdivision block graph is denoted by $\gamma_{ssb}(G)$. In this paper, some results on $\gamma_{ssb}(G)$ where obtained in terms of vertices, blocks and other parameters of G.

We need the following Theorems for our further results:

[II] MAIN RESULTS

Theorem A [4]: A split dominating set D of G is minimal for each vertex $v \in D$, one of the following condition holds.

i) There exists a vertex $u \in V - D$, such that $N(u) \cap D = \{v\}$.

ii) v is an isolated vertex in $\langle D\rangle$.

iii) $\langle (V - D) \cup \{v\}\rangle$ is connected.

Theorem B [4]: For any graph , $\gamma_s(G) \leq \frac{p.\Delta(G)}{1+\Delta(G)}$.

Now we consider the upper bound on $\gamma_{ssb}(G)$ in terms of blocks in G .

Theorem 2.1: For any graph G with $n - blocks$ *and* $n \geq 2$, then $\gamma_{ssb}(G) \leq n - 1$.

Proof: For any graph G with $n = 1$ $block$, a split domination does not exists. Hence we required $n \geq 2$ blocks. Let $S = \{B_1, B_2, B_3, \dots \dots \dots \dots B_n\}$ be the number of blocks of G and $M = \{b_1, b_2, b_3, \dots \dots \dots \dots b_n\}$ be the vertices in $B(G)$ with corresponding to the blocks of S. Also $V = \{v_1, v_2, v_3, \dots \dots \dots . v_n\}$ be the set of vertices in $[SB(G)]$. Let $V_1 = \{v_1, v_2, v_3, \dots \dots \dots . v_i\}$, $1 \leq i \leq n, V_1 \subset V$ be a set of cut vertices. Again consider a subset V_1^1 of V such that $\forall v_i \in N(V) \cap N(V_1^1) and\, V_1 = V - V_1^1$. Let $V_2 = \{v_1, v_2, v_3, \dots \dots \dots . v_s\}$,$1 \leq s \leq n$, $\forall v_s \in V$ which are not cut vertices such that $N(V_1) \cap N(V_2) = \emptyset$, $then$ $\{V_1 \cup V_2\}$ is a dominating set .Clearly $V[SB(G) - \{V_1 \cup V_2\} = H$ is disconnected graph. Then $(V_1 \cup V_2)$ is a $\gamma_{ssb} - set\ of\ G$. Hence $|V_1 \cup V_2| = \gamma_{ssb}(G)$ which gives $\gamma_{ssb}(G) \leq n - 1$.

In the following Theorem, we obtain an upper bound for $\gamma_{ssb}(G)$ in terms of vertices added to $B(G)$.

Theorem 2.2: For any connected (p, q) graph with $n \geq 2$ blocks, then $\gamma_{ssb}(G) \leq R$ where R is the number of vertices added to $B(G)$.

Proof: For any nontrivial connected graph G. If the graph G has $n = 1$ $block$.Then by the definition, split domination set does not exists. Hence $n \geq 2$ $blocks$.Let $S = \{B_1, B_2, B_3, \dots \dots \dots \dots B_n\}$ be the blocks of G and $M = \{b_1, b_2, b_3, \dots \dots \dots \dots b_n\}$ be the vertices in $B(G)$ which corresponds to the blocks of S.Now we consider the following cases.

Case1: Suppose each block of $B(G)$ is an edge. Then $R = q = E[B(G)]$. Let $V = \{v_1, v_2, v_3, \dots \dots \dots . v_n\}$ be the set of vertices of $[SB(G)]$. Now consider $V_1 = \{v_1, v_2, v_3, \dots \dots \dots . v_i\}$, $1 \leq i \leq n$ is a set of cut vertices in $[SB(G)]$.

$Let\ V_2 \subseteq V_1$, $\forall v_j \in V_2$ are adjacent to end vertices of $[SB(G)]$. Again there exists a subset V_3 of V_1 with the property $V[SB(G)] - \{V_2 \cup V_3\} = H$ where $\forall v_n \in H$ is adjacent to atleast one vertex of $(V_2 \cup V_3)$ and H is a disconnected graph . Hence $V_2 \cup V_3$ is a γ_{ssb} $set\ of\ G$. By Theorem 1,

$|V_2 \cup V_3| \leq R$.

Case2: Suppose each block of $B(G)$ is a complete graph with $p \geq 3$ $vertices$. Again we consider the sub cases of case 2.

Subcase2.1: Assume $B(G) = K_p$, $p \geq 3$. Then $V[SB(G)] = V[B(G)] + q[B(G)]$ and $V[SB(G)] - V[B(G)] = q[B(G)]$ where $\forall v_i \in q[B(G)]$ is an isolates. Hence $|q[B(G)]| \geq |V[B(G)]|$ which gives $\gamma_{ssb}(G) \leq R$.

Sub case 2.2: Assume every block of $B(G)$ is K_p,$p \geq 3$. . Let $B(G) = \{K_{p_1}, K_{p_2}, K_{p_3}, \dots \dots \dots K_{pm}\}$ then $V\{S[B_1(G) \cup B_2(G) \cup B_3(G) \dots \dots \dots . \cup B_m(G)]\} = V[B_1, B_2, B_3, \dots \dots \dots . B_m] + q_1[B(G)] \cup q_2[B(G)] \cup q_3[B(G)] \dots \dots \dots \cup q_m[B(G)]$
and $V\{S[B_1(G) \cup B_2(G) \cup B_3(G) \dots \dots \dots . \cup B_m(G)]\} - V[B_1, B_2, B_3, \dots \dots \dots . B_m] = q_1[B(G)] \cup q_2[B(G)] \cup q_3[B(G)] \dots \dots \dots \dots \cup q_m[B(G)]$. where $v_i \in q_1[B(G)] \cup q_2[B(G)] \cup q_3[B(G)] \dots \dots \dots \dots \cup q_m[B(G)]$ is an isolate. Hence $|q_1[B(G)] \cup q_2[B(G)] \cup q_3[B(G)] \dots \dots \dots \dots \cup q_m[B(G)]| \geq |V[B_1, B_2, B_3, \dots \dots \dots . B_m]|$ which gives $\gamma_{ssb}(G) \leq R$.

We establish an upper bound involving the Maximum degree $\Delta(G)$ and the vertices of G for split block sub division domination in graphs.

Theorem 2.3: For any graph G $with\ n \geq 2\ blocks$, then $\gamma_{ssb}(G) \leq \left\lfloor \frac{p.\Delta(G)}{1+\Delta(G)} \right\rfloor$.

Proof: For split domination, We consider the graphs with the property $n \geq 2$ $blocks$. Let $S = \{B_1, B_2, B_3, \dots \dots \dots \dots B_n\}$ be the blocks of G and $M = \{b_1, b_2, b_3, \dots \dots \dots \dots b_n\}$ be the vertices in $B(G)$ corresponding to the blocks of S. Let $V = \{v_1, v_2, v_3, \dots \dots \dots . v_n\}$ be the vertices in $[SB(G)]$. Let D be a $\gamma_s - set\ of\ [SB(G)]$. By Theorem A, each vertex $v \in D$,there exist a vertex $u \in V[SB(G)] - D$ is a split dominating set in $[SB(G)]$.Thus $\gamma(G) \leq |V[SB(G)] - D|$, $\gamma(G) \leq P - \gamma_{ssb}(G)$. Since by Theorem B, $\gamma_s(G) \leq \frac{p.\Delta(G)}{1+\Delta(G)}$ which gives $\gamma_{ssb}(G) \leq \left\lfloor \frac{p.\Delta(G)}{1+\Delta(G)} \right\rfloor$.

The following lower bound relationship is between split domination in $[SB(G)]$ and vertex covering number in $B(G)$.

Theorem 2.4: For any graph G with $n \geq 2$ *blocks* ,then $\gamma_{ssb}(G) \geq \alpha_0[B(G)]$, where α_0 is a vertex covering number of $B(G)$.

Proof: We consider only those graphs which are not $n = 1$. Let $S = \{B_1, B_2, B_3, \dots \dots \dots B_n\}$ be the blocks of G which correspondes to the set $M = \{b_1, b_2, b_3, \dots \dots \dots b_n\}$ be the vertices in B(G). Let $V = \{v_1, v_2, v_3, \dots \dots \dots . v_n\}$ be the vertices in $[SB(G)]$ *such that* $M \subset V$. Again $D = \{v_1, v_2, v_3, \dots \dots \dots . v_i\}$, $1 \leq i \leq n, D \subset V$ such that $N(v_i) \cap N(v_j) = v_k$,$v_i, v_j, \in D$ and $v_k \in V[SB(G)] - D$ *and* $N(v_i) \cap N(v_j) = \emptyset$, $\forall v_i, v_j, \in D$. Hence $\langle V[SB(G)] - D\rangle$ is disconnected, which gives $|V[SB(G) - D| = \gamma_{ssb}(G)$. Now $M_1 = \{b_1, b_2, b_3, \dots \dots \dots b_i\}, 1 \leq i \leq n$ and $M_1 \subset M$ and each edge in $B(G)$ is adjacent to atleast one vertex in M_1. Clearly $|M_1| = \alpha_0[B(G)]$. Hence $|V[SB(G)] - D| \geq |M_1|$ which gives $\gamma_{ssb}(G) \geq \alpha_0[B(G)]$.

The following result gives a upper bound for $\gamma_{ssb}(G)$ in terms of domination and end blocks in G.

Theorem 2.5: For any connected graph G with $n \geq 2$ *blocks* and $N -$ *end blocks*, then

$$\gamma_{ssb}(G) \leq \gamma(G) + N.$$

Proof: Suppose graph G is a block .Then by definition, the split domination does not exists. Now assume G is a graph with at least two blocks. Let $S = \{B_1, B_2, B_3, \dots \dots \dots B_n\}$ be the set of blocks in $G$and $M = \{b_1, b_2, b_3, \dots \dots \dots b_n\}$ be the vertices in $B(G)$ which corresponds to the blocks of G. Now $V = \{v_1, v_2, v_3, \dots \dots \dots . v_n\}$ be the vertices in $[SB(G)]$. Suppose D is a $\gamma_s - set$ *in*$[SB(G)]$ of G, whose vertex set is $V = \{v_1, v_2, v_3, \dots \dots \dots . v_i\}$. Note that at least one$v_i \in S$. More over, any component of $V - S$ is of size atleast two. Thus D is minimal which gives $|D| = \gamma_{ssb}(G)$. Again $S_1 = \{u_1, u_2, u_3, \dots \dots \dots . u_n\}$ be the vertices in G and $D_1 = \{u_1, u_2, u_3, \dots \dots \dots . u_i\}$,$1 \leq i \leq n, D_1 \subset S_1$. Every vertex of $S_1 - D_1$ is adjacent to at least one vertex of D_1. Suppose there exists a vertex $v \in D_1$ such that every vertex of $D_1 - V_1$ is not adjacent to at least one vertex $u \in [S_1 - \{D_1 - v\}]$. Thus $|S_1 - D_1| = \gamma(G)$. Hence $|D| \leq |S_1 - D_1| + N$ which gives $\gamma_{ssb}(G) \leq \gamma(G) + N$.

A relationship between the split domination in $[SB(G)]$ and independence number of a graph G is established in the following theorem.

Theorem2.6: For any connected graph G with $n \geq 2$ *blocks* then $\gamma_{ssb}(G) \geq \beta_0(G) - 1$, *where* $\beta_0(G)$ is the independence number of G.

Proof: By the definition of split domination, $n \neq 1$. Let $S = \{B_1, B_2, B_3, \dots \dots \dots B_n\}$ be the blocks of G which corresponds to the vertices of the set $M = \{b_1, b_2, b_3, \dots \dots \dots b_n\}$ in $B(G)$. Let $V = \{v_1, v_2, v_3, \dots \dots \dots . v_n\}$ be the vertices in $[SB(G)]$ such that $M \subset V$ .Let $H = \{v_1, v_2, v_3, \dots \dots \dots . v_s\}$ be the set of vertices in G. We have the following cases.

Case1: Suppose $B(G)$ is a tree. Let $V_1^1 = \{v_1, v_2, v_3, \dots \dots \dots . v_s\}$ are cut vertices in $[SB(G)]$. Again $V_1^{11} = \{v_1, v_2, v_3, \dots \dots \dots . v_t\}, 1 \leq t \leq s$ and $V_1^{11} \subset V_1^1$,were$\forall v_t \in V_1^{11}$. Then we consider V_2^1, V_3^1, V_4^1 , where $V_1^{11} = \{v_1, v_2, v_3, \dots \dots \dots . v_t\} = V_2^1 \cup V_3^1 \cup V_4^1$ with the property *that* $N(v_i) \cap N(v_j) = \emptyset, \forall v_i \in V_2^1$ and $\forall v_j \in V_3^1$ and V_4^1 is a set of all end vertices in$[SB(G)]$. Again $\langle V[SB(G)]\rangle = J$ where every $v \in J$ is an isolates.Thus $|V_1^{11}| = \gamma_{ssb}(G)$.

Case 2: Suppose $B(G)$ is not a tree. Again we consider sub cases of case 2

Subcases2.1: Assume $B(G)$ is a block. Then in $[SB(G)]$, $V[SB(G)] = V[B(G)] + \{K\}$,*where* $\forall$ *k*, *degk* $= 2$. Thus $|K| = P_0$ the number of isolates in $V[SB(G)] - V[B(G)]$. Hence $|V[B(G)]| = \gamma_{ssb}(G)$. One can see that for the $\beta_0 - set$ as in case1, We have $|V[B(G)]| \geq \beta_0 - 1$ which gives $\gamma_{ssb}(G) \geq \beta_0(G) - 1$.

Sub case 2.2: Assume $B(G)$ has atleast two blocks.Then as in subcase 2.1,we have $\gamma_{ssb}(G) \geq \beta_0(G) - 1$..

The next result gives a lower bound on $\gamma_{ssb}(G)$ in terms of the diameter of G.

Theorem 2.7: For any graph G with $n \geq 2$ blocks ,then $\gamma_{ssb}(G) \geq$ *diameter*$(G) - 2$.

Proof : Suppose $S = \{B_1, B_2, B_3, \dots \dots \dots B_n\}$ be the blocks of G,Then $M = \{b_1, b_2, b_3, \dots \dots \dots b_n\}$ be the corresponding block vertices in B(G). Suppose $A = \{e_1, e_2, e_3, \dots \dots \dots e_k\}$ be the set of edges which constitutes the diameteral path in G.Let $S_1 = \{B_i\}$ where $1 \le i \le n, S_1 \subset S$. Suppose $\forall B_i \in S_1$ are non end blocks in G, which gives cut vertices in $B(G)$ and $[SB(G)]$. Suppose $V = \{v_1, v_2, v_3, \dots \dots \dots . v_n\}$ be the vertices in $[SB(G)]. Again\ V_1 = \{v_1, v_2, v_3, \dots \dots \dots . v_i\}$ where $1 \le i \le n$ such that $V_1 \subset V\ then\ \forall v_i \in V_1$ are cut vertices in $[SB(G)]$. Since they are non end blocks in $[SB(G)]$. Then V_1 is a $\gamma_s - set\ of\ [SB(G)]$. Clearly $|V_1| = \gamma_{ssb}(G)$.

Suppose G is cyclic then there exists atleast one block B which contains a block diametrical path of length atleast two. In $B(G)$ the block $B \in V[B(G)]$ as a singleton and if atmost two elements of $\{A\} \notin$ diameter of G then $|A| - 2 \le |V_1|$ gives$\gamma_{ssb}(G) \ge diameter(G) - 2$. Suppose G is acyclic then each edge of G is a block of G. Now $\forall B_i \in S, \exists\ e_i, e_j \notin \{A\}$, where $1 \le \{i, j\} \le k$ gives $diameter\ (G) - 2 \le |V_1|$. Clearly we have $\gamma_{ssb}(G) \ge diameter(G) - 2$.

The following result is a relationship between $\gamma_{ssb}(G)$, domination and vertices of G.

Theorem 2.8: For any graph G with $n \ge 2\ blocks\ then\ \gamma_{ssb}(G) + \gamma(G) \le P + 1$.

Proof: Suppose the graph G has one block, then split domination does not exists. Hence $n \ge 2\ blocks$.

Suppose $S = \{B_1, B_2, B_3, \dots \dots \dots B_n\}$ be the blocks of G.Then $M = \{b_1, b_2, b_3, \dots \dots \dots b_n\}$ be the corresponding block vertices in $B(G)$. Let $H = \{v_1, v_2, v_3, \dots \dots \dots . v_n\}$ be the set of vertices in G. Also $J = \{v_1, v_2, v_3, \dots \dots \dots . v_i\}$ where $1 \le i \le n$ such that $J \subset H$ and $\forall v_i \in H - J$ is adjacent to atleast one vertex of $J. Hence\ |J| = \gamma(G). Let\ V = \{v_1, v_2, v_3, \dots \dots \dots . v_s\}$ be the set of vertices in $[SB(G)]$. Now $S_1 = \{B_i\}$ where $1 \le i \le n, S_1 \subset S\ and\ \forall B_i \in S_1$ are non end blocks in G. Then we have $V_1 \subset V$ which corresponds to the elements of $S[S_1]$ such that V_1 forms a minimal dominating set of $[SB(G)]$. Since each element of V_1 is a cut vertex, then $|V_1| = \gamma_{ssb}(G)$. Further $V_1 \cup J \le P + 1$ which gives $\gamma_{ssb}(G) + \gamma(G) \le P + 1$.

Next, the following upper bound for split domination in $[SB(G)]$ is interms of edge covering number of G.

Theorem2.9: For any connected (p, q) graph with $n \ge 2\ blocks, then\ \gamma_{ssb}(G) \le \alpha_1(G) + 1$ where $\alpha_1(G)$ is the edge covering number.

Proof: For any non trivial connected graph G with $n = 1\ block$, then by definition of split domination, the split domination set does not exists. Hence $n \ge 2\ blocks$.

Let $S = \{B_1, B_2, B_3, \dots \dots \dots B_n\}$ be the blocks of G which correspondes to the set $M = \{b_1, b_2, b_3, \dots \dots \dots b_n\}$ be the vertices in $B(G)$. Let $V = \{v_1, v_2, v_3, \dots \dots \dots . v_n\}$ be the vertices in $[SB(G)]$ such that $M \subset V$. We have the following cases.

Case 1: Suppose each block is an edge in G .Then $E(G) = |E_1(G) \cup E_2(G)|$ where $E_1(G)$ is the set of end edges, If every cut vertex of G is adjacent with an end vertex. Then $\exists\ E_1(G) and\ E_2(G). If\ E_2(G) = \emptyset$. Then $|E_1(G)| = \alpha_1(G)$ .Otherwise $|E_1(G) \cup E_2(G)| = \alpha_1(G)$.

Let $D_1 = \{v_s\}, 1 \le s \le n\ and\ D_1 \subset V$, then there exist atleast one cut vertices in $[SB(G)]$. Let $D_2 = \{v_t\}$ $1 \le t \le n, D_2 \subset V$ which are non cut vertices in$[SB(G)]$. Again $D_2^1 = \{v_l\}, 1 \le l \le t$ and $D_2^1 \subset D_2$. The $N(D_2^1) \cap N(v_s) = \emptyset\ then\ (D_2^1 \cup D_1)$ is a split dominating set. Hence $\langle V[SB(G)] - (D_2^1 \cup D_1)\rangle = \gamma_{ssb}(G)$. Since $\langle V[SB(G)] - (D_2^1 \cup D_1)\rangle$ has more than one component. Hence$\left|V[SB(G)] - (D_2^1 \cup D_1)\right| \le \alpha_1(G) + 1$ which gives $\gamma_{ssb}(G) \le \alpha_1(G) + 1$.

Case2: Suppose G has atleast one block which is not an edge. Let $D_1 = \{v_1, v_2, v_3, \dots \dots \dots . v_i\}, 1 \le i \le n$ and $D_1 \subset V$ be the set of cut vertices such that $N(v_i) \ne \emptyset$. Again $D_2 = \{v_1, v_2, v_3, \dots \dots \dots . v_l\}, 1 \le l \le i$ be the set of cut vertices in $[SB(G)]$ such that $N(v_i) \cap N(v_l) = \emptyset$ $, N(v_i) \cap N(v_l) = v_k, where\ v_i, v_l, \in D$ and $v_k \in V[SB(G)] - D$. Hence $\langle V[SB(G)] - D\rangle$ is disconnected , which gives $|V[SB(G)] - D| = \gamma_{ssb}(G)$. As in case 1 , $\alpha_1(G)$ will increase. Hence $|V[SB(G)] - D| \le \alpha_1(G) + 1$ which gives $\alpha_1(G) + 1 \ge \gamma_{ssb}(G)$.

The following lower bound for split domination in $[SB(G)]$ is interms of edge independence number in $B(G)$.

Theorem 2.10: For any graph G with $n \geq 2$ *blocks then* $\gamma_{ssb}(G) \geq \beta_1[B(G)]$.

Proof: By the definition of Split domination, we need $n \geq 2$ *blocks*. We have the following cases.

Case 1: Suppose each block in $B(G)$ is an edge. Let $E = \{e_1, e_2, e_3, \dots\dots\dots . e_n\}$ be the set of edges in $B(G)$. Also $E_1 = \{e_s\}, 1 \leq s \leq n$ be a set of alternative edges in $B(G)$. Then $|E_1| = \beta_1[B(G)]$.

Consider $V = \{v_1, v_2, v_3, \dots\dots\dots . v_n\}$ be the vertices in $[SB(G)]$, again $V_1 = \{v_1, v_2, v_3, \dots\dots\dots . v_i\}$ be the cut vertices which are adjacent to at least one vertex of E_1 and $V_2 = \{v_s\}$ are the end vertices in $[SB(G)]$. *Further* $\langle V[SB(G)] - (V_1 \cup V_2)\rangle$ is disconnected. Then $|V_1 \cup V_2|$ *is a* γ_{ssb} *− set.*

Hence $|V_1 \cup V_2| \geq |E_1|$ which gives $\gamma_{ssb}(G) \geq \beta_1[B(G)]$.

Case2: Suppose there exists at least one block which is not an edge. Let $E = \{e_1, e_2, e_3, \dots\dots\dots . e_n\}$ be the set of edges in $B(G)$. Again $E_1 = \{e_s\}, 1 \leq s \leq n$ is the set of alternative edges in $B(G)$ which gives $|E_1| = \beta_1[B(G)]$.

Suppose $V = \{v_1, v_2, v_3, \dots\dots\dots . v_n\}$ be the vertices of $[SB(G)]$. Then $V = V_1 \cup V_2$ where V_1 is a set of cut vertices and V_2 is a set of non cut vertices. Now we consider $V_1^1 \subset V_1$ *and* $V_2^1 \subset V_2$ such that $\langle V[SB(G)] - (V_1^1 \cup V_2^1)\rangle$ has more than one component. Hence $\{V_1^1 \cup V_2^1\}$ is a γ_{ssb} *− set* and $|V_1^1 \cup V_2^1| \geq \beta_1[B(G)]$ which gives $\gamma_{ssb}(G) \geq \beta_1[B(G)]$.

In the following theorem, we expressed the lower bound for $\gamma_{ssb}(G)$ in terms of cut vertices of $B(G)$.

Theorem 2.11: For any connected graph G with $n \geq 2$ *blocks* then $\gamma_{ssb}(G) \geq C[B(G)]$ where C is the cut vertices in $B(G)$.

Proof: Suppose graph G is a block. Then by the definition, of split domination, $n \geq 2$. consider the following cases.

Case 1: Suppose each block of $B(G)$ is an edge. Then we consider $S = \{v_1, v_2, v_3, \dots\dots\dots . v_m\}$ be the cut vertices in $B(G)$. *Now* $V = \{v_1, v_2, v_3, \dots\dots\dots . v_n\}$ be the vertices in $[SB(G)]$ and $V_1 = \{v_i\}$ $1 \leq i \leq n$ are cut vertices in $[SB(G)]$. *Again* $V_2 \subset V_1$ is adjacent to at least one vertex in S. Then $V[SB(G)] - V_2$ gives disconnected graph. Thus $|V_2| = \gamma_{ssb}(G)$. *Hence* $|V_2| \geq C[B(G)]$ gives $\gamma_{ssb}(G) \geq C[B(G)]$.

Case 2: Suppose each block in $B(G)$ is not an edge. Let $S_1 = \{v_1, v_2, v_3, \dots\dots\dots . v_s\}$ be the cut vertices in $[SB(G)]$. *Then* $S_1 \cong S$. Again $S_2 = \{v_1, v_2, v_3, \dots\dots\dots . v_l\}$ are the non cut vertices in $[SB(G)]$.Further we consider $S_2^1 \subset S_2$ such that $V[SB(G)] - \{S_2^1\} \cup \{S\} = H$ where $\langle H\rangle$ is disconnected. Clearly $|S_2^1 \cup S_1| \geq |S|$ which gives $\gamma_{ssb}(G) \geq C[B(G)]$.

Finally, the following result gives an lower bound on $\gamma_{ssb}(G)$ in terms of $\gamma_{cot}(G)$.

Theorem 2.12: For any nontrivial tree with $n \geq 2$ *blocks*, $\gamma_{ssb}(G) \geq \gamma_{cot}(G) - 1$.

Proof: We consider only those graphs which are not $n = 1$. Let $H = \{v_1, v_2, v_3, \dots\dots\dots . v_p\}$, $H_1 = \{v_1, v_2, v_3, \dots\dots\dots . v_i\}$, $1 \leq i \leq p$ be a subset of $V(G) = H$ which are end vertices in G. Let $J = \{v_1, v_2, v_3, \dots\dots\dots . v_j\} \subseteq V(G)$ with $1 \leq j \leq p$ such that $\forall v_j \in J$, $N(v_i) \cap N(v_j) = \emptyset$ and $\langle V(G) - (H_1 \cup J)\rangle$ has no isolates, then$|H_1 \cup J| = \gamma_{cot}(G)$. Let $V = \{v_1, v_2, v_3, \dots\dots\dots . v_n\}$ be the vertices in $[SB(G)]$. consider $D = \{v_1, v_2, v_3, \dots\dots\dots . v_t\} = V_1 \cup V_2 \cup V_3$ be the set of all vertices of$[SB(G)]$. *Where* $\forall v_s \in V_1$ *and* $v_t \in V_2$ with the property $(v_s) \cap N(v_t) = \emptyset$, $\forall v_l \in V_3$ is a set of all end vertices in $[SB(G)]$. *The* $\langle D\rangle$ *is an isolates* . $|D|$ gives minimum split domination in $[SB(G)]$.

Thus $|D| = \gamma_{ssb}(G)$. Clearly $|H_1 \cup J| - 1 \leq |D|$ which gives $\gamma_{ssb}(G) \geq \gamma_{cot}(G) - 1$.

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