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PROFIT ANALYSIS IN SOME REDUNDANT SYSTEMS WITH REPAIR MAINTENANCE

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Abstract— **This paper deals with a modularly redundant system with many active units and a warm standby unit. The concepts of 'coverage' and 'manual recovery' have been incorporated. Probabilities that the system can recover automatically/manually at the time of failure of an active unit, are fixed. Failure time distributions of an active and standby units are exponential with different rates. However, distributions of time to repair a failed unit, recovery device, time to manual recovery pre taken as general. It is assumed that the system earns a fixed amount for the duration it is operative and repair cost is incurred when a unit/RD is under repair. Expected profit of the system has been obtained by superimposing Howard's reward structure on the semi-Markov process generated by the system model. System performance (expected profit) has been studied for its behaviour. Several earlier well known models are included as special cases.**

Keywords— **Cloud computing, Multi-tenancy, Virtualization, Cloud resource monitoring, simulation.**

I. INTRODUCTION

Expected profit is an extremely important parameter in economic evaluation of standby redundant systems. In fact, the environmen s under which modern complex business/industrial standby systems operate are critically economic sensitive. A review of uhe existing literature on standby systems reveals that economic aspects have not been analysed to the satisfactory extent. Most of the authors were interested in obtaining LS transform of the first passage distribution to system failure', availability of a system2' 3.

Recently expected profit has been obtained for a two-dissimilar unit system4 and has been suggesj ed as the measure of maintenance effectiveness'. Optimal preventive maintenance policies that maximize expected profit rate in a two-unit standby system with degraded states has also been discussed by Mine Kawai6. Switch behaviour has also been incorporated in the evaluation of profit in a 2-unit warm standby redundant system.

The present paper deals with a system consisting of several units with a common warm standby. Con-cepts of `coverage'7 have also been incorporated. System performance (expected profit) has been related with other parameters e.g., failure rates of a unit, repair-time disc ribution of a failed unit, earning rate of the sys-tem, repair cost etc. The purpose of the paper is to discuss following aspects of standby redundant systems. (i) To obtain analytic expression for the expected profit, the system will earn in steady-state if it is allowed to operate in an infinite time span. (ii) To investigate the response of expected profit to changes in other system parameters viz., mean-time to failure, mean-time to repair, earning rate of the system etc. (iii) To examine the impact of 'coverage' and 'manual recovery' on the economics of the system. (*,v) To study the effec' of the warm standby on expected profit. The model discussed is quite general and includes several earlier well known models as special cases, some of them'are shown in the end. For the purpose of analysis, an income-structure' has been superimposed on the semi-Markov process generated by the system model.

II. SYSTEM MODEL

There is a (n+1) unit sysf-em., n units are required to operate in order to perform the necessary system task and one unit is put in the common warm standby. A warm standby can fail while as standby. (ii) Failure-time •distributions of operative and standby units are exponential whereas repair-time distribution is general. (iii) There are following two devices : (a) Automatic Recovery Device : It is used to switch the standby unit if it is there) to operate at the time of failure of an operative unit. (b) Manual Recovery Device : Some faults are not covered by ARD but a manual action may recover the system without performing the actual repair.

bability of ARD operating successfully at the time of need is fixed. Probability that a fault can be r manually is also fixed. When ARD fails, it goes to repair immediately and the failed unit -wails foe"veT, cause of a single repairman. Distribution of time to repair ARD is general. Further, time taken t Pair 3 system manually is also random with general distribution. recover (iv) Units and ARD are like new ones after each repair. (V) The system earns (looses) a fixed amount per unit time in each state and transition rewards (cosi 8) are involved whenever it changes its state. (Vi) All random variables defined to model the system and independent in statistical sense. le system model allows different failure rates for an operative and standby units which is required in eleeiro-c and power systems. By giving priority to repair ARD, system down-time will be reduced which will result in increased profit.

III. SYSTEM STATES AND TRANSITIONS

Define the following system states to identify the system at any time. : n units are operative and a unit is as warm standby, 82 : a unit is under repair and the system is operational after successful recovery, 83 : MRD is under repair, 84 ARD is under repair and the failed unit is waiting for repair, S5 : one unit is under repair and another failed unit is waiting for repair. Initially, Aystem starts in Si. Upon failure of active unit, ARD is used t o recover the system's task i.e., switch the standby unit to operate; if ARD is successful, system enters i,('2 but if ARD does not operate -..operly, system may be recovered manually in which it enters again S2. But if MRD is not good system goes S3. Transitions between states are shown in Fig. 1. System is up in 81, S2 and it is down in S_3, S_4, S_5.

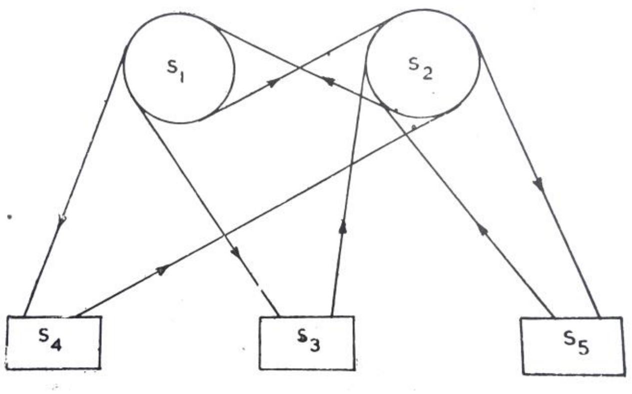

Fig. 1 Transition diagram for the model.

IV. NOTATION

λ constant failure rates for an operative unit
λ1 constant hazard rate for standby unit
ρ ≡ λ1/ n λ normalised value of hazard rate
P probability that the system can recover automatically given that an active unit has failed
υ probability that the f, ystem can be recovered manually but not automatically (0⩽ u ⩽ p).
f(t) p.cl,f. of rep air-tune of a failed unit
fl(t) p.d.f. of repair-time for ARD
f2 (t) p.d.f. for repair-time for MRD
Φ Laplace transform of f(t) evaluated at n λ
m expected time to repair a failed unit
f(t) p.cl,f. of rep air-tune of a failed unit
fl(t) p.d.f. of repair-time for ARD
f2 (t) p.d.f. for repair-time for MRD
Φ Laplace transform of f(t) evaluated at n λ
m expected time to repair a failed unit
mean-time to repair ARD
m2 mean-time to repair MRD
M,M1,M2 , ≡ mn λ , M1 ,n λ , M2 , n λ
μi mean unconditional sojourn time of the system in Si
Pij one-step transition probability from Si to Sj
P transition probability matrix , ≡(pij)
I identity matrix of order 5
D I —P
Di subdeberminent of D, deleting ith row and ith column
πi probability that the embedded Markov chain is in Si, ≡ di/∑_i▒di
rii transition reward for a transition from Si to Sj
λ i earning rate per i/nA time of the system in Si
g expected profit per 1/nA time in steady-state
implies the complement e.g, Φ = 1 — Φ.

V. ANALYSIS OF RESULTS

It has been shown in Howard (1964) that

$$g = \sum \pi_i \, \mu_i \, q_i \Big/ \sum \pi_i \, \mu_i$$

$$\mu_i \, q_i = \sum_i p_{ij} \, r_{ij} + y_i \, \mu_i \, n\lambda$$

It may be easy to see that the semi-Markov process generated by the system is irreducible. Elements of P are given by

$$p_{12} = p \int_0^\infty e^{-\lambda_1 t} \, n\lambda e^{-n\lambda t} \, dt + \int_0^\infty e^{-n\lambda t} \, \lambda_1 e^{-\lambda_1 t} \, dt = (p + \rho)/(1 -$$

$$p_{14} = (1 - p - u) \int_0^\infty e^{-\lambda_1 t} \, n\lambda e^{-n\lambda t} \, dt = (1 - p - u)/(1 + \rho),$$

$$p_{13} = u \int_0^\infty e^{-\lambda_1 t} \, n\lambda e^{-n\lambda t} \, dt = u/(1 + \rho),$$

$$p_{25} = \int_0^\infty n\lambda e^{-n\lambda t} \, \bar{F}(t) \, dt = \bar{\phi},$$

$$p_{21} = \int_0^\infty e^{-n\lambda t} \, dF(t) = \phi,$$

$$p_{32} = p_{41} = p_{52} = 1, \text{ and}$$
$$p_{ij} = 0 \text{ for other } i \text{ and } j.$$

Further, we can also find

$$d_1 = \phi \ , \ d_2 = 1 \ , \ d_3 = u\phi/(1+\rho) \ , \ d_4 = (1-p-u)\ \phi/(1+\rho)$$

$$d_5 = \overline{\phi} \ , \ \mu_1 = \int_0^\infty e^{-n\lambda t} e^{-\lambda_1 t}\, dt = 1/n\lambda\,(1+\rho)\ ,$$

$$\mu_2 = \int_0^\infty e^{-n\lambda t}\,\overline{F}(t)\, dt = \overline{\phi}/n\lambda,\ \mu_3 = m_1,\ \mu_4 = m_2,$$

$$\mu_5 = (M - \overline{\phi})/n\lambda\overline{\phi}\ ,$$

Substituting above into (1) and simplifying, we get

g = W

where

$$W \equiv \Big[(p+\rho)\, r_{12} + u\,(r_{13} + r_{32} + y_3 M_1) + (1-p-u)\,(r_{14} + r_{42} + y_4 M_4{}^6) + + y_1 \Big] \phi + (1+\rho) \Big[(r_{25} + r_{52} + y_2 - y_5)\,\overline{\phi} + r_{21}\,\phi + y_5 M \Big]$$

$$X = (1+\rho)\, M + \Big[1 + uM_1 + (1-p-u)\, M_2 \Big] \phi$$

Particular Cases (i) If u = 0, n = 1, then (3) reduces to where

y = W/X

Where,

$$W \equiv \Big[(p+\rho)\, r_{12} + q\,(r_{14} + r_{42} + y_4 M_2) + y_1 \Big] \phi + + (1+\rho) \Big[(r_{25} + r_{52} + y_2 - y_5)\,\overline{\phi} + r_{21}\,\phi + y_5 M \Big]$$

$$X \equiv (C+\rho)\, M + (1 + qM_2)\,\phi$$

$$q = 1 - p$$

The above result is in agreement with equation (2) in kumar5 for the case when f(t) = f1(t).

Further let us consider the following cost structure :

R: earnings of the system per 1/ λ times when system is operative

CI: repair cost per 1/ λ times for a failed unit when it is under

Ca: repair repair cost per 1/A tinies for AIM to be repaired. So, substituting y1=R, y2=R-C,y4= -Cd ,y5= -C, rij =0 for all i & j into (3), We get

G=W/X

……….(4)

Where ,

$$W \equiv R(1 + \rho\overline{\phi}) - \Big[(1+\rho)\, MC + q\,\phi\, M_2\, C_d \Big]$$

$$X \equiv (1+\rho)\, M + (1 + M_2 q)\,\phi$$

Obviously, g given by (4) is a non-decreasing function of R and a non-increasing finction of C and Cd. In order to examine the effect of warm redundancy on expected profit for a cold standby case i.e. put p = (4) to get profit for a c

Where

$$g_0 = W_0/X_0$$

$$W_0 \equiv R - (MC + qM_2\, C_d)$$

$$X_0 \equiv M + (1 + qM_2)$$

So loss is expected profit due to failure of a unit while in standby is given by

L= g — go = W/X ……(6)

$$W \equiv \rho \Big[R \Big\{ M - \overline{\phi}\,(M + (1 + qM_2)\,\phi) \Big\} + M \Big\{ C\,(1 + qM_2)\,\phi - qM_2\, C_d\, \phi \Big\} \Big]$$

$$X \equiv \Big[M + (1 + qM_2)\,\phi \Big] \Big[(1+\rho)\, M + (1 + qM_2)\,\phi \Big]$$

It is evident from the above equation that loss vanishes if p 0. Also, function of Cd and is a non-decreasing function of C.

(ii) If p = 1, u = 0, n = 1, then (2) reduced to

g = W/X

Where

$$W \equiv \phi \Big[(r_{12} + r_{21})\,\phi + r_{21} + y_1 \Big] + (1+\rho) \Big[(r_{25} + r_{52} + y_2 - y_5)\,\overline{\phi} + y_5 M \Big]$$

$$X \equiv (1+\rho)\, M + \phi$$

This agrees with (2) in Kumar' for ϕs =ϕ0, ms = mo.

(iii) If p = 1, u = 0, n = 1, λo = nλ+λi and λ' = nλ the model reduces to a 2unit parallel redundant system'. In this case (2) reduces to

g = W/X

where

$$W \equiv (1+\rho) \Big[(r_{01} + r_{10})\,\phi + (r_{12} + r_{21} + y_1)\,\overline{\phi} + (M - \overline{\phi})\, y_2 \Big] + y_0\,\phi$$

$$X \equiv \phi + (1+\rho)\, M$$

$$\rho = \lambda_1/\lambda'$$

In the above paper Nakagawa & ()saki' have included four earlier well known mo(So, those models can easily be derived as special cases of the present special case. So given by Gaver[2] and Downton.[9]

Conclusions

We ba-ve, obtained expected profit for a modularly redundant system. Model contains several earlier well known 'models as special cases. Concepts of automatic and manual recovery incorporated in the model arequite useful parameters to system designers. Probabilities p and u are just design parameters and it is upt-o system designers to examine what constitutes these proportions in their cases.

`Coverage' is defined as the proportion of faults from which a system can recover automatically7. This proporcion could really be controlled to the maximum possible extent. However, a line has to be imposed between recoverable and non-recoverable faults and the overall situation be examined either from the view point of objective functions or economics of the situation. Recoverable faults are usually connected with the software or the programming part of computer systems and non-recoverable faults are attributed to the hardware design portion. The concept of 'Black Box' explains the limits under which automatic coverage is economically feasible. It will not be out of place to mention that adaptive systems basically make no distinc-tion between recoverable and non-recoverable failure states.

In order for a coverage to be complete and exhaustive two fundamental conditions in terms of concepts of 'Black Box' must be satisfied.

(i) The instrumental data must be complete and sufficient to define the situation completely,

(ii) The mathematical model must be capable of getting the solution. A.s we go on moving towards the so called 'complete strategy', marginal cost increases rapidly and therefore a Line separating one from the other (recoverable and non-rec overable) would solely depend upon objective functions.

Above discussion defines completely the concept of automatic recovery or 'coverage', Hence 'coverage' may be defined as a 'strategy'. to recover from certain undesirable states within economic constraints and Without supply of any data trom the outside world.

The impact of automatic recovery, manual recovery, warm redundancy etc. on the overall economics of the system must be considered well in advance.

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