PORFIT IN 2-UNIT STANDBY SYSTEM WITH A GENERAL COST STRUCTURE

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Abstract— **A 2- unit cold standby system with exponential failure time and general repair time distribution is considered . A quantity of great relevance in the maintenance of equipments in modern business and industrial systems , systems , vzt., profit that system will earn per unit time , has been obtained analytically under unidiscounted and discounted cost structure by superimposing Howard's reward structure on the semi-Markov generated by the system. Steady – state profit has also been studied for its behaviour for a special case.**

Index Terms— **cold standby system, great relevance, obtained analytically under unidiscounted.**

I. INTRODUCTION

Parameters of interested in 2-unit standby repairable systems have been obtained by different workers. They have used semi-Markov process1, regenerative process2, Markov renewal process3 to obtain mean time to system failure, steady-state availability, expected number of visits to a state in a given time t etc.

The environments under which maintained standby systems operate are 'critically economic'. A review pf the existing literature on standby redundant systems reveals that economic aspects have not extensively been investigated so for. A quantity of immense importance in problems of maintenance and replacement of equipment's in modern business and industrial systems is the expected profit that system earns per unit time. Recently Ashok kumar4 has obtained steady-system profit in a 2- unit warm standby system under undiscounted cost- structure and has suggested optimal preventive maintenance policies that maximize expected profit5. However, the total profit we expect a system to earn in time t, if it starts to operate at time t=0, in still more important and gives information of economic behaviour of a standby redundant system. In the present paper Howard's 6 reward structure has been superimposed on the semi – Markov process generated by a 2-unit cold standby system to obtain profit under undiscounted as well as discounted cost structures.

II. SYSTEM MODEL

1. There is a 2-unit cold standby redundant system, i.e. a unit does not fail in standby interval.

2. Failure time distribution of an operative unit is exponential and is independent of repair time distribution which is assumed to be general.

3. Each unit can be in one of the states :

 o: operating , s: standby , r: repair , wr: wait for repair.
We define the following system states:

 S0: (o, s), S1: (o, r), S2: (wr, r)
The system is up in S0 and S1 and the system is down is S2.
Transitions between states are given below:

 From S0 can go to S1

 From S1 can go to S0 to S2

 From S2 can go to S1

4. There is only one repairman and service discipline is 'first come, first served'.

5. Switching is perfect; instantaneous and without errors.

6. System earns (looses) at a fixed rate in each state, which can differ From state to state. There is a transition reward (cast) whenever the system changes state. This can be different for each transition.

III. NATATIONS

λ	Constant failure rate of operative unit
I,j	Subscripts which imply system states: 0,1,2.
p_{ij}	1 – step transition probability from S_i to S_j
P	Transition probability matrix (p_{ij})
I	Identity matrix of order 3
G(t)	*Cdf* of repair time
$\emptyset$(s)	Denotes laplace transform of G (t), evaluated as s
–	Implies the complement e.g., $\emptyset = (1 - \emptyset)$
M	Mean repair time
m =	$m\lambda$
μi	Unconditional expected value of sojourn time in Si
r_{ij}	Reward for a transition from S_i to S_j
λ_{yi}	Amount that the system earns per unit time in Si
λg	Expected net profit of the system per unit time
Z_i (t)	Total profit system would earn in time t, If at t= 0 , it was in Si

Z (t)   Column vector $(Z_1(t), Z_2(t), Z_3(t))$

g_i Earning rate of the system in S_i

β Exponential discounting rate

$f_{ij}(t)$ pd$f$ of the wait for the system in S_i given that the next transition is to S_j

$q_{ij}(t)$ $= p_{ij} f_{ij}(t)$

$h_i(t)$ pdf of the waiting time of the system in S_i
  $= \sum_j p_{ij} f_{ij}(t) = \sum_j q_{ij}(t)$

$\emptyset_{ij}$ Laplace transform pf $q_{ij}(t)$ evaluated at β

q_i Laplace transform pf $h_i(t)$ evaluated at β

δ Matrix $\emptyset_{ij}$

η_i Present value of the expected reward that will be obtained when the system occupies S_i and upon its departure from S_i

$(I-\delta)^{-1}$ Denotes inverse matrix of $(I-\delta)$

η Column vector (η_1, η_2, η_3)

IV. SYSTEM CHARACTERISTICS

It is easy obtain the following parameters directly [1] by putting $\lambda_2 = 0$

$p_{01} = 1, p_{10} = \emptyset(\lambda), p_{12} = \emptyset(\lambda), p_{21} = 1$

$P_{ij} = 0$,for all other I and j.

$\mu_0 = 1/\lambda$, $\mu_1 = 1/\lambda$, $\mu_2 = \{M - \emptyset(\lambda)\}/\lambda\emptyset(\lambda)$

$f_{01}(t) = \lambda e^{-\lambda t}$, $f_{10}(t) = \lambda e^{-\lambda t} g(t)/\emptyset(\lambda)$,

$f_{12}(t) = \lambda e^{-\lambda t} G(t)/\emptyset(\lambda)$, $f_{21}(t) = \lambda e^{-\lambda t} t\int \infty\, e^{-\lambda y} g(y)\, dy / = \emptyset(\lambda)$

Further, by taking Laplace transform of the required functions and taking inverse of the matrix (I-δ), we get following additional parameters of interest .

$$\phi_{01} = \lambda/(\lambda + \beta), \quad \phi_{.0} = 1/(\lambda + \beta),$$

$$\phi_{1:} = \lambda\{1 - \phi(\lambda + \beta)\}/(\lambda + \beta),$$

$$\phi_{.3} = \lambda\{\phi(\lambda) - \phi(\beta)\}/(\beta - \lambda)\,\bar{\phi}(\lambda)$$

$$(I - \delta)^{-1} = \frac{1}{(1 - \phi_{01}\phi_{10})} \begin{bmatrix} (1 - \phi_{12}\phi_{.1} & \phi_{0:} & \phi_{0}\cdot\phi_{1:} \\ -(\phi_{10}\phi_{01} + \phi_{12}\phi_{.1})/\phi_{01} & 1 & \phi_{1:} \\ \phi_{21}(1 - \phi_{.01}\phi_{.0})/\phi_{01} & 0 & (1 - \phi_{01}\phi_{10}) \end{bmatrix}$$

Unidiscount reward structure and system profits. It has been shown in Howard [6] that for large values of t.

$$Z_i(t) = v_i + \lambda g t \tag{1}$$

$$v_i + \lambda g\,\mu_i = g_i\,\mu_i \tag{2}$$

$$g_i\mu_i = \sum_j p_{ij} r_{ij} + \lambda y_i\,\mu_i , \tag{2a}$$

Where v_1 is the transient part of the profit and gt is condition of the steady – state behaviour of the system. The growth rate g is a quantity which we call profit of the system . thus profit is the average reward per unit time the system will earn in steady – state.

We shall use (2) as the basic eqution to evaluate system profit . substitution the required values in (2) we get

$$v_0 - v_1 + g\lambda\mu_0 = r_{01} + \lambda y_0\mu_0 \tag{3}$$

$$\phi(\lambda)\,v_0 + v_1 - \bar{\phi}(\lambda)\,v_2 + \lambda g\mu_1 = r_{10}\,\phi(\lambda) + r_{12}\,\bar{\phi}(\lambda) + \lambda y_1\,\mu_1 \tag{4}$$

$$-v_1 + v_2 + g\lambda\mu_2 = r_{21} + \lambda y_2\,\mu_2 . \tag{5}$$

These are 3 equtions in 4 unknowns viz., v0 ,v1 and g. we observe that addition of a constant to all vi's does not alter the equations. Hence one vi can arbitrarily be designated, leaving 3 equation in 3 unknowns. Setting v2 =0 and solving (3),(4) and (5). We get

$$[\phi(\lambda) + M]\,g = (r_{01} + r_{10} + y_0)\,\phi(\lambda) + (r_{12} + r_{21} + y_1 - y_2)\,\bar{\phi}(\lambda) + My_2 \tag{6}$$

$$\bar{\phi}(\lambda)\,v_0 = (r_{01} - r_{21}\,y_0)\,\bar{\phi}(\lambda) - (M - \bar{\phi}(\lambda))\,y_2 - 2\,[\bar{\phi}(\lambda) - M]g \tag{7}$$

$$\bar{\phi}(\lambda)v_1 = (g - y_2)\,(M - \bar{\phi}(\lambda)) - r_{21}\,\bar{\phi}(\lambda) \tag{8}$$

In rest of this section we concentrate on expected profit g only. Consider now exponential repair- distribution and the following cost structure,

Y: earning rate of the system per $1/\lambda$ time when it is operating

C: repair cost rate per $1/\lambda$ time of a failed unit

So , substitute y0 = y, y1= y-c, rij =0 for I and j and $\emptyset(\lambda) = 1/(1+M)$ into (6) , to get

$$g = \frac{(y - cM)(1 + M)}{1 + M\,(1 + M)} \tag{9}$$

To investigate the properties of expected profit given by (9) we require following lemma.

Lemma
  Let,

$$f(x) = \frac{ax^2 + bx + d}{a'x^2 + b'x + d'}$$

Be a function defined on a compact set [0, xi] .
let A= ab'-ba', B= 2(ad'-da') , D= bd' – db'. Then

$f(x)$ is non – increasing function of x if either (a) if A> 0 then D<0, $Ax_1^2 + bx_1 + D <0$ or (b) if A<0 then D<0 , $Ax_1^2 + bx_1 + D < 0$
And $B(2 Ax_1 + B) > 0$

Proof

$$\frac{df}{dx} = \frac{(a'x^2 + b'x + d')(2ax+b) - (ax^2+bx+d)(2a'x+b')}{(a'x^2 + b'x + a')^2}$$

Now

N= numerator of df/dx is

$$x^2(ab' - ba') + 2x(aa' - da') + (ba' - db')$$
$$= Ax^2 + Bx + D \qquad (11)$$

$X2(ab'-ba')+2x(ad'-da')+ bd' - db'$
$= Ax2+Bx+D$

For f(x) to be non – increasing a sufficient condition is

$$\frac{df(x)}{dx} \leqslant 0 \qquad (12)$$

$Ax2+Bx+D=0$

Consider the quardratic eqution
$Ax2+Bx+D=0$

Case (i)

If A>0 then (14) represented a parabola of the form as shown in /fig.1
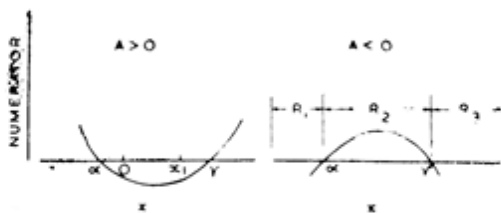

Fig. !—Behaviour of $Ax^2 + Bx + D = 0$ when $A > 0$ and $A < 0$

Fig. 1- behavior of Ax2+Bx+D =0 when a>0 and A<0

Then for (13) to hold , both the extreme points of [0,x] must lie between (α,γ) as otherwise function will be positive at some points .

So, at x= 0, $Ax2+Bx+D \leqslant 0$ implies $D \leqslant 0$;
And at x=x1 , $Ax2+Bx+D \leqslant 0$ implies $Ax2+Bx+D \leqslant 0$

Case (ii)

If A< 0 then (14) is of the form as shown in 'fig.1.

In this case for (13) to hold good the interval [0, x1] should lie outside (α , γ) . in other words , either [0,x1] $\in$ R1 or [0,x1] $\in$ R3 .

$$\text{Slop of} \quad Ax^2 + Bx + D = 0 \text{ is } m = 2Ax + B \qquad (10)$$
$$\text{at} \quad x = 0, m = m_0 = B; \text{ at } x = x_1, m = m_1 = 2Ax_1 + B$$
$$\text{Also} \quad D \leqslant 0, \text{ at } x = 0 \text{ and } Ax_1^2 + Bx_1 + D \leqslant 0, \text{ at } x = x_1$$
$$\text{So} \quad m_0 m_1 > 0 \text{ or } B(2Ax_1 + B) > 0$$

Slop of $Ax^2+Bx+D =0$ is m= 2 Ax^1 +B
At x=0, $m=m_0$ =B; at x=x1 , $m=m_1 = 2Ax_1$ +B
Also $D \geq$, at x=0 and $Ax_1^2 +Bx_1+D \leq 0$, at x= X_1
So $m_0m_1>0$ or $B(2 Ax^1 +B) >0$
Hence the lemma , follows .

Some of the properties of g given by (9) are given below :

g>0 for $M \leq y/c$, i.e. , system runs in profit it rraffic intensity is less than or equal to ratio of the normalized earing rate to normalized repair cost .
g is monotonicallu non – increasing function of M . this can be proved by applying the lemma .
now a= -c ,b=y-c , d=y, a'= 1 , d'= 1
A=-y , B=2(-c-y), D=-c
and A<0, D<0 , Ax2+Bx+D =0 . so the conditions (i) are satisfied and thus the property follows .
max g= y
$0 \leq M \leq y/c$
Min g= 0
$0 \leq M \leq y/c$

V. DISCOUNTED MODEL

In some industrial application whose span covrs large periods of time , we must discount any payment in the future to some parent value. We have assumed exponential discounting rate $\beta>0$,i.e. , payments recived at time t in the future are considered at time t to ve worth e-βt as much now .

the effect of discounting on the profit is that it losses its linearly increasing nature . when discounting is used . the present value of the stream of payments expected from the system in an infinite time is finite for any starting state Si. it has been shown 6 that

$$\eta_i = \Sigma_j r_{ij} \phi_{ij} + \{\lambda y_i (1 - \phi_i)/\beta\}$$
$$Z(\infty) = (I - \delta)^{-1} \eta$$

for the present models

$$\eta_1 \beta = r_{01}\, \phi_{01}\, \beta \;+\; \lambda y_0\, \bar{\phi}_{01}$$

$$\eta_2 \beta = (r_{10}\, \phi_{.0} + r_{12}\, \phi_{12})\, \beta \;+\; y_1 \lambda \,\{1 - (\phi_{10} + \phi_{12})\}$$

$$\eta_2 \beta = r_{21}\, \phi_{21}\, \beta \;+\; y_2 \lambda \bar{\phi}_{21}$$

from (15) to (19) , we get after simplification

$$Z_0\,(\infty) = N/D,$$

Where

$$N = \frac{\lambda\,(r_{01} + y_0)}{\xi\theta} \left[\xi\theta - \lambda' \bar{\phi}\,(\lambda) \left\{ \phi\,(\lambda) - \phi\,(\beta) \right\} \right] +$$

$$+ \frac{\lambda}{\xi}\left[\xi\, r_{10}\, \phi\,(\xi) + \lambda \bar{\phi}\,(\xi)\,(r_{12} + y_1) \right] +$$

$$+ \frac{\lambda^3 \bar{\phi}\,(\xi)}{\xi\beta\theta}\left[\beta\, r_{21} \left\{ \phi\,(\lambda) - \phi\,(\xi) \right\} + y_2 \left\{ \theta - \lambda\,(\phi(\lambda) - \phi\,(\xi)) \right\} \right] \quad (21)$$

$$D = \lambda \bar{\phi}\,(\xi) + \beta \quad (22)$$

$$\xi = (\beta + \lambda),\ \theta = (\beta - \lambda)$$

CONCLUSION

We have obtained expressions for profit in a 2- unit cold – stand by redundant system under undiscount ed and discounted cost structure . steady – state profit has also been studied for ius behaviour for a particular case . these parametes are quite useful in economic evaluation of stand by redundant systems. A few application areas from defence are given below :

1. in any production system engaged in manufacturing of components used in HF-24 , the failure of power even for a few seconds may be economically critical and therefor a generator may be kept as standby which will be energized at the time of power failure .

2. in air-borne radar systems a wrong detection can cost heavily to an aircrafat. So standby radar systems may be used to ensure high reliability of the system.

3. In the management of refuellers used at airports to provide fuel to aircrafts , even a delay by a few moments will upset the complet schedule of flights . so a stand by arrangements may be desirable.

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