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ON SOME FRACTIONAL INTEGRAL FORMULAE INVOLVING THE MULTIVARIABLE I-FUNCTION AND A GENERAL CLASS OF POLYNOMIALS

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Abstract— **In the present paper, the authors have obtained two fractioal integral formulae involving the product of a general class of polynomials and the multivariable I-function. On account of the most general nature of these functions, a number of results (known and new) follow as special cases of our formulae. In the end, we obtain a fractional integral formula involving the laguerre and the hermite polynomials as a simple special case of our main formula.**

Keywords- **Riemann-Liouville Fractional Integral Operator, Multivariable I-function, General Class of Polynomials, Mellin-Barnes contour Integral**

(2000 Mathematics subject classification: 33C99).

I. Introduction

In the present paper the fractional integral operator will be defined and represented as follows:

$$ {}_cI_x^v\left\{f(x)\right\}=\frac{1}{\Gamma(v)}\int_c^x (x-t)^{v-1}f(t)dt, \quad \mathrm{Re}(v)>0 $$

(1.1)

The special case of the above fractional integral for $c=0$ is the Riemann-Liouville fractional integral operator and will be written as $I_x^v\left\{f(x)\right\}$.

Also, the fractional integral operator investigated by Erdelyi-Kober will be defined and represented as follows:

$$ I_x^{\eta,v}\left\{f(x)\right\}=\frac{x^{-\eta-v+1}}{\Gamma(v)}\int_0^x (x-t)^{v-1}t^{\eta-1}f(t)dt \ , \mathrm{Re}(v)>0, \eta>0 $$

(1.2)

Clearly, this is a generalization of Riemann-Liouville fractional integral operator.

The multivariable I-function introduced by Prasad [2] will be define and represent it in the following manner :

$$ I\left[z_1,\dots,z_r\right]=I^{0,n_2:\dots:0,n_r:(m',n'):\dots:\left(m^{(r)},n^{(r)}\right)}_{p_2,q_2:\dots:p_r,q_r:(p',q'):\dots:\left(p^{(r)},q^{(r)}\right)} $$

$$ \left[z_1,\dots,z_r\left|\begin{matrix}(a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\dots:(\alpha_{rj},\alpha'_{rj},\dots,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'}:\dots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}}\\(b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\dots:(b_{rj},\beta'_{rj},\dots,\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'}:\dots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}}\end{matrix}\right.\right] $$

$$ =\frac{1}{(2\pi w)^r}\int_{L_1}\dots\int_{L_r}\phi_1(s_1)\dots\phi_r(s_r)\psi(s_1,\dots,s_r)z_1^{s_1},\dots,z_r^{s_r}ds_1\dots ds_r $$

(1.3)

Where

$$ w=\sqrt{(-1)} $$

$$ \phi_i(s_i)=\frac{\prod_{j=1}^{m^{(i)}}\Gamma\left(b_j^{(i)}-\beta_j^{(i)}s_i\right)\prod_{j=1}^{n^{(i)}}\Gamma\left(1-a_j^{(i)}+\alpha_j^{(i)}s_i\right)}{\prod_{j=m^{(i)}+1}^{q^{(i)}}\Gamma\left(1-b_j^{(i)}+\beta_j^{(i)}s_i\right)\prod_{j=n^{(i)}+1}^{p^{(i)}}\Gamma\left(a_j^{(i)}-\alpha_j^{(i)}s_i\right)} \quad \forall i\in(1,2,\dots,r) $$

(1.4)

$$ \psi(s_1,\dots,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma\left(1-a_{2j}+\sum_{i=1}^{2}\alpha_{2j}^{(i)}s_i\right)\prod_{j=1}^{n_3}\Gamma\left(1-a_{3j}+\sum_{i=1}^{3}\alpha_{3j}^{(i)}s_i\right)}{\prod_{j=n_2+1}^{p_2}\Gamma\left(a_{2j}-\sum_{i=1}^{2}\alpha_{2j}^{(i)}s_i\right)\prod_{j=n_3+1}^{p_3}\Gamma\left(a_{3j}-\sum_{i=1}^{3}\alpha_{3j}^{(i)}s_i\right)} $$

$$ \frac{\dots\prod_{j=1}^{n_r}\Gamma\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i\right)}{\dots\prod_{j=n_r+1}^{p_r}\Gamma\left(a_{rj}-\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i\right)\prod_{j=1}^{q_2}\Gamma\left(1-b_{2j}-\sum_{i=1}^{2}\beta_{2j}^{(i)}s_i\right)\dots\prod_{j=1}^{q_r}\Gamma\left(1-b_{rj}-\sum_{i=1}^{r}\beta_{rj}^{(i)}s_i\right)} $$

(1.5)

$\alpha_j^{(i)}, \beta_j^{(i)}, \alpha_{kj}^{(i)}, \beta_{kj}^{(i)} (i=1,...,r)(k=1,...,r)$ are positive numbers,
$a_j^{(i)}, b_j^{(i)} (i=1,...,r), a_{kj}, b_{kj} (k=2,...,r)$ are complex numbers and here
$m^{(i)}, n^{(i)}, p^{(i)}, q^{(i)} (i=1,...,r), n_k, p_k, q_k (k=2,...,r)$ are non-negative integers where $0 \le m^{(i)} \le q^{(i)}, 0 \le n^{(i)} \le p^{(i)}, q^k \ge 0, 0 \le n_k \le p_k$. Here (i) denotes the numbers of dashes. The contours L_i in the complex s_i -plane is of the Mellin-Barnes type which runs from $-w\infty$ to $+w\infty$ with indentations, if necessary, to ensure that all the poles of $\Gamma\left(b_j^{(i)} - \beta_j^{(i)} s_i\right)\left(j=1,...,m^{(i)}\right)$ are separated from those of $\Gamma\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)} s_i\right)\left(j=1,...,n_r\right)$.

For further details and asymptotic expansion of the I -function one can refer by Prasad [2].

In what follows, the multivariable I -function defined by [2] will be represented in the contracted notation:

$$I^{0,n_2:...:0,n_r:(m',n'):...:(m^{(r)},n^{(r)})}_{p_2,q_2:...:p_r,q_r:(p',q'):...:(p^{(r)},q^{(r)})}\left[z_1,...,z_r\right]$$

Or simply by $I\left[z_1,...z_r\right]$.

According to the asymptotic expansion of the gamma function, the counter integral (1.3) is absolutely convergent provided that

$$\left|\arg z_i\right| < \frac{1}{2}\pi U_i, U_i > 0 \quad ; \quad i=1, 2, \; ....., r \tag{1.6}$$

Where

$$U_i = \sum_{j=1}^{n^i}\alpha_j^{(i)} - \sum_{j=n^{(i)}+1}^{p^{(i)}}\alpha_j^{(i)} + \sum_{j=1}^{m^i}\beta_j^{(i)} - \sum_{j=m^{(i)}+1}^{q^{(i)}}\beta_j^{(i)}$$

$$+(\sum_{j=1}^{n_2}\alpha_{2j}^{(i)} - \sum_{j=n_2+1}^{p_2}\alpha_{2j}^{(i)}) + (\sum_{j=1}^{n_3}\alpha_{3j}^{(i)} - \sum_{j=n_3+1}^{p_3}\alpha_{3j}^{(i)})$$

$$+...+(\sum_{j=1}^{n_r}\alpha_{rj}^{(i)} - \sum_{j=n_r+1}^{p_r}\alpha_{rj}^{(i)})$$

$$-(\sum_{j=1}^{q_2}\beta_{2j}^{(i)} + \sum_{j=1}^{q_3}\beta_{3j}^{(i)} + ... + \sum_{j=1}^{q_r}\beta_{rj}^{(i)}) \tag{1.7}$$

The asymptotic expansion of the I -function has been discussed by Prasad [2]. His results run as follow:

$$I[z_1,...,z_r] = 0(|z_1|^{\alpha_1}...|z_r|^{\alpha_r}), \max\{|z_1|,...,|z_r|\} \to 0$$

Where

$\alpha_i = \min \mathrm{Re}(b_j^{(i)} / \beta_j^{(i)})$, $j=1, \; ....., m^{(i)}$; $i=1, \; ....., r$

And

$$I[z_1,...,z_r] = 0(|z_1|^{\beta_1}...|z_r|^{\beta_r}), \min\{|z_1|,...,|z_r|\} \to \infty$$

Where

$\beta_i = \max \mathrm{Re}(\frac{a_j^{(i)}-1}{\alpha_j^{(i)}})$; $j=1, \; ....., n^{(i)}$, $i=1, \; ....., r$

The details of the function can be found in the paper of Prasad [2].

Again we shall use the following notation:

$$I^{0,n_2:...:0,n_r+u:...}_{p_2,q_2:...:p_r+u,q_r+v:...}\left[\begin{array}{c|c} z_1 & \left(\rho_j;\lambda_j^{'},...,\lambda_j^{(r)}\right)_{1,u},... \\ \vdots & \\ z_r & \left(\sigma_j;v_j^{'},...,v_j^{(r)}\right)_{1,v},... \end{array}\right]$$

For the multivariable I -function given by (1.3)

Srivastava ([4], p.1, eq. (1)) introduced a general class of polynomials $S_n^m[x]$ defined by means of the following equation:

$$S_n^m[x] = \sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}x^k, \quad n=0,1,2,... \tag{1.8}$$

Where m is an arbitrary positive integer and the coefficients $A_{n,k}(n,k \ge 0)$ are arbitrary constants, real or complex. On specializing these coefficints $A_{n,k}; S_n^m$ yields a number of known polynomials as special cases. These include, among others, the Hermite polynomials, the Jacobi polynomials, the Laguerre polynomials, the Bessel polynomials, the Gould-Hopper polynomials, the Brafman polynomials and many others.

II. Fractional Integral Formulae

$${}_cI_x^{\nu}\left\{x^{\rho}(x+\alpha)^{\sigma}(x+\beta)^{\mu}S_n^m\left[ax^u(x+\alpha)^v(x+\beta)^w\right]\right.$$

$$S_{n'}^{m'}\left[bx^{u'}(x+\alpha)^{v'}(x+\beta)^{w'}\right]I\left[z_1x^{u_1}(x+\alpha)^{v_1}(x+\beta)^{w_1},...,\right.$$

$$\left.\left.z_rx^{u_r}(x+\alpha)^{v_r}(x+\beta)^{w_r}\right]\right\}$$

$=$

$$\alpha^{\sigma}\beta^{\mu}x^{\rho}\sum_{s,l,t=0}^{\infty}\sum_{k=0}^{[n/m]}\sum_{k'=0}^{[n'/m']}(-n)_{mk}(-n')_{m'k'}A_{n,k}A'_{n',k'}a^kb^{k'}$$

$$\alpha^{vk+v'k'-1}\beta^{wk+w'k'-1}\frac{(-1)^s(x-c)^{s+v}x^{uk+u'k'+l+t-s}}{k!k'!l!t!\Gamma(v)\,(s+v)s!}$$

$$I^{0,n_2:...:0,n_r+3:...}_{p_2,q_2:...:p_r+3,q_r+3:...}\left[\begin{matrix} z_1\alpha^{v_1}\beta^{w_1}x^{u_1} \\ \vdots \\ z_r\alpha^{v_r}\beta^{w_r}x^{u_r}\end{matrix}\left|\begin{matrix}(-\rho-uk-u'k'-l-t;u_1,...,u_r), \\ \\ (s-\rho-uk-u'k'-l-t;u_1,...,u_r),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(-\sigma-vk-v'k';v_1,...,v_r),(-\mu-wk-w'k';w_1,...,w_r);... \\ (l-\sigma-vk-v'k';v_1,...,v_r),(t-\mu-wk-w'k';w_1,...,w_r);...\end{matrix}\right]$$

(2.1)

Provided that

(i)

$$\mathrm{Re}(v)>0;\min\{u_i,v_i,w_i,u,v,w,u',v',w'\}>0,i=1,2,...,r;$$

$$\max\left\{|\arg\left(\frac{x}{\alpha}\right),|\arg\left(\frac{x}{\beta}\right)\right\}<\pi$$

and the I-function of r variables involved in (2.1) satisfy the usual convergence and existance conditions.

$$\mathrm{Re}(\rho)+\sum_{i=1}^{r}u_i\min_{1\le j\le m^{(i)}}\left\{\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right\}+1>0$$

(ii)

(iii) The series occuring on the right-hand side of (2.1) is absolutely convergent.

$$I_x^{\eta,v}\left\{x^\rho(x+\alpha)^\sigma(x+\beta)^\mu S_n^m\left[ax^u(x+\alpha)^v(x+\beta)^w\right]\right.$$

$$S_{n'}^{m'}\left[bx^{u'}(x+\alpha)^{v'}(x+\beta)^{w'}\right]I\left[z_1x^{u_1}(x+\alpha)^{v_1}(x+\beta)^{w_1},...,\right.$$

$$\left.\left.z_rx^{u_r}(x+\alpha)^{v_r}(x+\beta)^{w_r}\right]\right\}$$

=

$$\alpha^\sigma\beta^\mu x^\rho\sum_{l,t=0}^{\infty}\sum_{k=0}^{\infty}\sum_{k=0}^{[n/m]}\sum_{k'=0}^{[n'/m']}(-n)_{mk}(-n')_{m'k'}A_{n,k}A'_{n',k'}a^kb^{k'}$$

$$\alpha^{vk+v'k'-1}\beta^{wk+w'k'-1}\frac{(-1)^s x^{uk+u'k'+l+t}}{k!k'!l!t!}$$

$$I^{0,n_2:...:0,n_r+3:...}_{p_2,q_2:...:p_r+3,q_r+3:...}\left[\begin{matrix} z_1\alpha^{v_1}\beta^{w_1}x^{u_1} \\ \vdots \\ z_r\alpha^{v_r}\beta^{w_r}x^{u_r}\end{matrix}\left|\begin{matrix}(1-\eta-\rho-uk-u'k'-l-t;u_1,...,u_r), \\ \\ (-v+1-\eta-\rho-uk-u'k'-l-t;u_1,...,u_r),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(-\sigma-vk-v'k';v_1,...,v_r),(-\mu-wk-w'k';w_1,...,w_r);... \\ (l-\sigma-vk-v'k';v_1,...,v_r),(t-\mu-wk-w'k';w_1,...,w_r);...\end{matrix}\right]$$

(2.2)

Where

$$\eta>0,\mathrm{Re}(\rho)+\sum_{i=1}^{r}u_i\min_{1\le j\le m^{(i)}}\left\{\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right\}>-\eta$$

The series occuring on the right-hand side of (2.2) is absolutely convergent and the set of conditions (i) specified in the formula (2.1) are satisfied.

Proof of (2.1):

To establish the fractional integral formula (2.1), we first express the general class of polynomials $S_n^m[x]$ and $S_{n'}^{m'}[x]$ occuring on its left-hand side in the series form given by (1.8) and replace the multivariable I-function occuring therein by its well known Mellin-Barnes contour integral [5]. The left-hand side of (2.1) (say Δ) takes the following form:

$$\Delta={}_cI_x^v\left\{x^\rho(x+\alpha)^\sigma(x+\beta)^\mu\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\right.$$

$$\left[ax^u(x+\alpha)^v(x+\beta)^w\right]^k\sum_{k'=0}^{[n'/m']}\frac{(-n')_{m'k'}}{k'!}A_{n',k'}$$

$$\left[bx^{u'}(x+\alpha)^{v'}(x+\beta)^{w'}\right]^{k'}\frac{1}{(2\pi i)^r}\int_{L_1}...\int_{L_r}\psi(\xi_1,...,\xi_r)$$

$$\phi_1(\xi_1)....\phi_r(\xi_r)\left[z_1x^{u_1}(x+\alpha)^{v_1}(x+\beta)^{w_1}\right]^{\xi_1}$$

$$\left.\left[z_rx^{u_r}(x+\alpha)^{v_r}(x+\beta)^{w_r}\right]^{\xi_r}d\xi_1...d\xi_r\right\}$$

(2.3)

Now interchanging the order of ξ_i-integrals and the fractional integral involved in (2.3), collecting powers of $x,(x+\alpha)$ and $(x+\beta)$ in the expression thus obtained, making use of the following binomial expressions for $(x+\alpha)^\sigma$ and $(x+\beta)^\mu$

$$(x+\alpha)^\sigma=\alpha^\sigma\sum_{l=0}^{\infty}\binom{\sigma}{l}\left(\frac{x}{\alpha}\right)^l;|\left(\frac{x}{\alpha}\right)|<1$$

$$(x+\beta)^\mu=\beta^\mu\sum_{t=0}^{\infty}\binom{\mu}{t}\left(\frac{x}{\beta}\right)^t;|\left(\frac{x}{\beta}\right)|<1$$

And changing the order of ξ_i-integrals and l,t-series, which is easily seen to be permissible under conditions stated, we get after a little simplification the following result:

$$\Delta=\sum_{l,t=0}^{\infty}\sum_{k=0}^{[n/m]}\sum_{k'=0}^{[n'/m']}\frac{(-n)_{mk}}{k!}\frac{(-n')_{m'k'}}{k'!}A_{n,k}A'_{n',k'}$$

$$\frac{\alpha^{\sigma+vk+v'k'-l}\beta^{\mu+wk+w'k'-t}}{l!k!}\frac{1}{(2\pi i)^r}\int_{L_1}...\int_{L_r}\psi(\xi_1,...,\xi_r)$$

$$\phi_1(\xi_1)....\phi_r(\xi_r)\left[z_1\alpha^{v_1}\beta^{w_1}\right]^{\xi_1}...\left[z_r\alpha^{v_r}\beta^{w_r}\right]^{\xi_r}$$

$${}_cI_x^v\left\{x^{\rho+uk+u'k'+u_1\xi_1+...+u_r\xi_r+l+t}\right\}\frac{\Gamma\left(\sigma+vk+v'k'+v_1\xi_1+...+v_r\xi_r+1\right)}{\Gamma\left(\sigma+vk+v'k'+v_1\xi_1+...+v_r\xi_r-l+1\right)}$$

$$\frac{\Gamma\left(\mu+wk+w'k'+w_1\xi_1+...+w_r\xi_r+1\right)}{\Gamma\left(\mu+wk+w'k'+w_1\xi_1+...+w_r\xi_r-t+1\right)}d\xi_1...d\xi_r$$

(2.4)

Now making use of the familiar formula ([],p.22)

$$ {}_cI_x^{\nu}\left\{x^{\lambda}\right\} = \sum_{s=0}^{\infty} \frac{(-1)^s \Gamma\left(\lambda+1\right)(x-c)^{s+\nu} x^{\lambda-s}}{\Gamma(\nu)\Gamma\left(\lambda-s+1\right)\left(s+\nu\right)s!},\ \mathrm{Re}(\lambda) > -1 \tag{2.5} $$

In eq. (2.5), changing the order of s-series and ξ_i-integrals, which is also permissible under conditions stated and interpreting the resulting multiple Mellin-Barnes contour integral so obtained in terms of I-function of r variables, we finally arrive at the desired result.

In order to establish the fractional integral formula (2.2) (involving Erdelyi-Kober operator defind by (1.2), we proceed along the lines followed in (2.1) and make use of the result

$$ I_x^{\eta,\nu}\left\{x^{\lambda}\right\} = \frac{\Gamma\left(\lambda+\eta\right)x^{\lambda}}{\Gamma\left(\lambda+\eta+\nu\right)},\ \mathrm{Re}(\lambda) > -\eta \tag{2.6} $$

In place of formula (2.5).

3. Special Cases

Each of the fractional integral formulae (2.1) and (2.2) has two-fold generality. Not only do the general class of polynomials involved in these results reduce, under special cases, to a large spectrum of polynomials defined by Srivastava [4], the multivariable I-function occuring herein can also be suitably specilized to a remarkable wide variety of useful functions. A number of results (known and new) follow as special cases of these formulae.

If in (2.1) and (2.2), we put $n_2 = \ldots = n_{r-1} = 0 = p_2 = \ldots = p_{r-1}, q_2 = \ldots = q_{r-1} = 0$, the multivariable I-function reduces to multivariable H-function and we get result given by Gupta et. al. [1]

$$ {}_cI_x^{\nu}\left\{x^{\rho}(x+\alpha)^{\sigma}(x+\beta)^{\mu}\, S_n^m\left[ax^u(x+\alpha)^v(x+\beta)^w\right] S_{n'}^{m'}\left[bx^{u'}(x+\alpha)^{v'}(x+\beta)^{w'}\right] H\left[z_1x^{u_1}(x+\alpha)^{v_1}(x+\beta)^{w_1},\ldots, z_rx^{u_r}(x+\alpha)^{v_r}(x+\beta)^{w_r}\right]\right\} $$

$$ = \alpha^{\sigma}\beta^{\mu}x^{\rho}\sum_{s,l,t=0}^{\infty}\sum_{k=0}^{[n/m]}\sum_{k'=0}^{[n'/m']}(-n)_{mk}(-n')_{m'k'}A_{n,k}A'_{n',k'}a^kb^{k'} \alpha^{vk+v'k'-1}\beta^{wk+w'k'-1}\frac{(-1)^s(x-c)^{s+\nu}x^{uk+u'k'+l+t-s}}{k!k'!l!t!\Gamma(\nu)\,(s+\nu)s!} $$

$$ H_{p+3,q+3:\ldots}^{n+3:\ldots}\left[\begin{matrix} z_1\alpha^{v_1}\beta^{w_1}x^{u_1} \\ \vdots \\ z_r\alpha^{v_r}\beta^{w_r}x^{u_r}\end{matrix}\left|\begin{matrix}(-\rho-uk-u'k'-l-t;u_1,\ldots,u_r),(-\sigma-vk-v'k';v_1,\ldots,v_r),(-\mu-wk-w'k';w_1,\ldots,w_r);\ldots \\ \\ (s-\rho-uk-u'k'-l-t;u_1,\ldots,u_r),(l-\sigma-vk-v'k';v_1,\ldots,v_r),(t-\mu-wk-w'k';w_1,\ldots,w_r);\ldots\end{matrix}\right.\right] \tag{3.1} $$

Provided all the conditions are satisfied given in (2.1), [1].

$$ I_x^{\eta,\nu}\left\{x^{\rho}(x+\alpha)^{\sigma}(x+\beta)^{\mu}\, S_n^m\left[ax^u(x+\alpha)^v(x+\beta)^w\right] S_{n'}^{m'}\left[bx^{u'}(x+\alpha)^{v'}(x+\beta)^{w'}\right] H\left[z_1x^{u_1}(x+\alpha)^{v_1}(x+\beta)^{w_1},\ldots, z_rx^{u_r}(x+\alpha)^{v_r}(x+\beta)^{w_r}\right]\right\} $$

$$ = \alpha^{\sigma}\beta^{\mu}x^{\rho}\sum_{l,t=0}^{\infty}\sum_{k=0}^{\infty}\sum_{k=0}^{[n/m]}\sum_{k'=0}^{[n'/m']}(-n)_{mk}(-n')_{m'k'}A_{n,k}A'_{n',k'}a^kb^{k'} \alpha^{vk+v'k'-1}\beta^{wk+w'k'-1}\frac{(-1)^s x^{uk+u'k'+l+t}}{k!k'!l!t!} $$

$$ H_{p+3,q+3:\ldots}^{n+3:\ldots}\left[\begin{matrix} z_1\alpha^{v_1}\beta^{w_1}x^{u_1} \\ \vdots \\ z_r\alpha^{v_r}\beta^{w_r}x^{u_r}\end{matrix}\left|\begin{matrix}(1-\eta-\rho-uk-u'k'-l-t;u_1,\ldots,u_r),(-\sigma-vk-v'k';v_1,\ldots,v_r),(-\mu-wk-w'k';w_1,\ldots,w_r);\ldots \\ \\ (-\nu+1-\eta-\rho-uk-u'k'-l-t;u_1,\ldots,u_r),(l-\sigma-vk-v'k';v_1,\ldots,v_r),(t-\mu-wk-w'k';w_1,\ldots,w_r);\ldots\end{matrix}\right.\right] \tag{3.2} $$

Provided all the conditions are satisfied given in (2.2), [1].

If we take $c = 0$ in (3.1) and make use of the following result:

$$ \sum_{s=0}^{\infty}\frac{(-1)^s}{\Gamma(s)(s+\nu)s!}H_{p,q+1:*}^{0,n:*}\left[\begin{matrix} z_1 \\ \vdots \\ z_r\end{matrix}\left|\begin{matrix}\left(a_j;\alpha_j',\ldots,\alpha_j^{(r)}\right)_{1,p}:* \\ (s-k;u_1,\ldots,u_r),\left(b_j;\beta_j',\ldots,\beta_j^{(r)}\right)_{1,q}:*\end{matrix}\right.\right] = H_{p,q+1:*}^{0,n:*}\left[\begin{matrix} z_1 \\ \vdots \\ z_r\end{matrix}\left|\begin{matrix}\left(a_j;\alpha_j',\ldots,\alpha_j^{(r)}\right)_{1,p}:* \\ (-\nu-k;u_1,\ldots,u_r),\left(b_j;\beta_j',\ldots,\beta_j^{(r)}\right)_{1,q}:*\end{matrix}\right.\right] $$

We arrive at the following fractional integral formula after a little simplification:

$$ {}_cI_x^{\nu}\left\{x^{\rho}(x+\alpha)^{\sigma}(x+\beta)^{\mu}\, S_n^m\left[ax^u(x+\alpha)^v(x+\beta)^w\right] S_{n'}^{m'}\left[bx^{u'}(x+\alpha)^{v'}(x+\beta)^{w'}\right] H\left[z_1x^{u_1}(x+\alpha)^{v_1}(x+\beta)^{w_1},\ldots, z_rx^{u_r}(x+\alpha)^{v_r}(x+\beta)^{w_r}\right]\right\} $$

$$ = \alpha^{\sigma}\beta^{\mu}x^{\rho}\sum_{l,t=0}^{\infty}\sum_{k=0}^{[n/m]}\sum_{k'=0}^{[n'/m']}(-n)_{mk}(-n')_{m'k'}A_{n,k}A'_{n',k'}a^kb^{k'} \alpha^{vk+v'k'-1}\beta^{wk+w'k'-1}\frac{(-1)^s(x-c)^{s+\nu}x^{uk+u'k'+l+t}}{k!k'!l!t!} $$

$$ H_{p+3,q+3:\ldots}^{n+3:\ldots}\left[\begin{matrix} z_1\alpha^{v_1}\beta^{w_1}x^{u_1} \\ \vdots \\ z_r\alpha^{v_r}\beta^{w_r}x^{u_r}\end{matrix}\left|\begin{matrix}(-\rho-uk-u'k'-l-t;u_1,\ldots,u_r),(-\sigma-vk-v'k';v_1,\ldots,v_r),(-\mu-wk-w'k';w_1,\ldots,w_r);\ldots \\ \\ (-\nu-\rho-uk-u'k'-l-t;u_1,\ldots,u_r),(l-\sigma-vk-v'k';v_1,\ldots,v_r),(t-\mu-wk-w'k';w_1,\ldots,w_r);\ldots\end{matrix}\right.\right] \tag{3.3} $$

Where the conditions of validity directly obtainable from those mentioned with (2.1) are satisfied.

If we take $a=u=1, v=w=0, m=1, A_{n,k}=\binom{n+\gamma}{n}\frac{1}{(\gamma+1)_k}$ and further take $b=u'=1, v'=w'=0, m'=2, A'_{n',k'}=(-1)^{k'}$ in (3.3), $S_n^m[x]$ occuring therein breaks into the Laguerre polynomials([],p.158,eq.(1.4)) and the integral formula (3.3) takes the following:

$$
{}_cI_x^{\nu}\left\{x^{(\rho+n')/2}(x+\alpha)^{\sigma}(x+\beta)^{\mu}L_n^{\gamma}(x)H_n'\left(\frac{1}{2\sqrt{x}}\right)\right.
H\Big[z_1x^{u_1}(x+\alpha)^{v_1}(x+\beta)^{w_1},...,
\left. z_rx^{u_r}(x+\alpha)^{v_r}(x+\beta)^{w_r}\Big]\right\}
$$

$$
= \alpha^{\sigma}\beta^{\mu}x^{\rho+\nu}\sum_{l,t=0}^{\infty}\sum_{k=0}^{n}\sum_{k'=0}^{[n'/2]}(-n)_k(-n')_{2k'}\binom{n+\gamma}{n}\frac{1}{(\gamma+1)_k}
(-1)^{k'}\alpha^{-l}\beta^{-t}x^{k+k'+l+t}
H_{p+3,q+3:...}^{n+3:...}\left[\begin{matrix} z_1\alpha^{v_1}\beta^{w_1}x^{u_1}\\ \vdots \\ z_r\alpha^{v_r}\beta^{w_r}x^{u_r}\end{matrix}\middle|\begin{matrix}(-\rho-k-k'-l-t;u_1,...,u_r),(-\sigma;v_1,...,v_r),(-\mu;w_1,...,w_r);...\\ (-\nu-\rho-uk-u'k'-l-t;u_1,...,u_r),(l-\sigma;v_1,...,v_r),(t-\mu;w_1,...,w_r);...\end{matrix}\right]
$$

(3.4)

The conditions of validity of (3.3) can be obtained directly from (2.1).

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