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LOSS COMPENSATION TECHNIQUES OF METAMATERIAL NEGATIVE REFRACTIVE INDEX LENS (MNRI-LENS)

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***Abstract*—Loss in metamaterial negative refractive lens is a major obstacle that limits its practical deployment in important applications such as sub-diffraction imaging, focusing & superlensing. Because of higher intrinsic loss, there has been intensified research in this area to find solution to this problem. Although initial theoretical and experimental efforts have shown various techniques to compensate these losses but most researches are still limited to theory and numerical simulations and only few experiments have been performed to validate the results. This paper discusses several schemes of loss compensation in metamaterial negative refractive index (M-NRI) lens. The formula for transmission loss has been established for flat uniaxial M-NRI lens of thickness d, by using Drude-Lorentz model of permittivity (ε_z) and permeability (μ_z).The relationship between transmission loss and S21 has also been derived. The actual M-NRI lens, in a realistic configuration, has yet to be developed.**

***Index Terms*—Metamaterials (MTM), Negative refractive index (NRI), dispersion, Dual negative materials (DNG), Left handed materials (LHM), Surface Plasmon, Metamaterials negative refractive index (M-NRI).**

I. INTRODUCTION

In the recent years there has been much interest in utilizing metamaterials negative refractive index (M-NRI) lens in optical and microwave applications. This M-NRI lens offers the possibility of designing novel optical and microwave devices that can be used to derive interesting properties e.g.

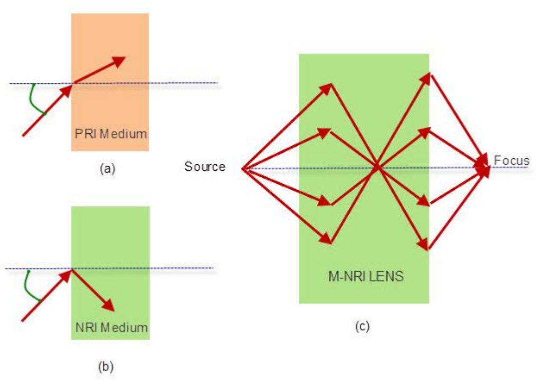

Fig. 1. (a) Refraction of ray in Positive Refractive Index (PRI) medium ; (b) Refraction of ray in Positive Refractive Index (PRI) medium (c) Focusing by flat metamaterial negative refractive index lens.

focusing from flat lens, sub-wavelength imaging,. The controlled anisotropy of M-NRI lens may also be used to tailor invisibility cloaks, directive antennas, and perfect lenses that overcome the diffraction limit. [1] [2]

Beside these aforementioned advantages, the intrinsic losses inherent in these materials plague the entire field of M-NRI lens and are one of the major restrictions preventing it from leaving the academic domain of research and entering the industrial applications.

In this paper, losses in M-NRI medium are discussed both numerically and analytically. The formula for transmission loss has been established for flat uniaxial M-NRI flat lens slab of thickness (d) using Drude-Lorentz model of permittivity (ε_z) and permeability (μ_z). The relationship between transmission loss and S21 has also been derived. Experimental and simulation results reported so far; show that the amount of loss would increase as the operating frequency increases. The transmission loss at lower frequencies is small (~ 1-5 dB/λ) and it becomes more severe at higher frequencies (up to ~ 30 dB/λ at infrared frequencies). Consequently, it is needed to study the loss and loss compensation techniques of M-NRI lens so that the interesting properties of this M-NRI lens can be utilized for industrial applications.

II. THEORY OF METAMATERIAL LENS

Most of M-NRI lens medium are analyzed using Lorentz-Drude response model [3]. For uniaxial NRI medium the permittivity and permeability tensors are defined as:

$$\varepsilon = \varepsilon_0 \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \tag{1}$$

$$\mu = \mu_0 \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_z \end{bmatrix} \tag{2}$$

In such a medium, the fields along optical axis of M-NRI lens are different from the fields transverse to it. These conditions give the following relationships:

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_T \neq \varepsilon_z \tag{3}$$

$$\mu_{xx} = \mu_{yy} = \mu_T \neq \mu_z \tag{4}$$

The Drude- Lorentz model is used to define frequency dependent permittivity and permeability of anisotropic medium:

$$\varepsilon_z = 1 - \frac{\omega_{pe}^2}{\omega^2 + j\omega\xi_e} = 1 - \frac{\omega_{pe}^2}{\omega^2 + \xi_e{}^2} + j\frac{\xi\omega_{pe}^2}{\omega(\omega^2 + \xi_e{}^2)} \tag{5}$$

$\omega_{pe} \rightarrow$ Electric plasma frequency (Non resonant);

$\xi_e \rightarrow$ Damping factor due to metal loss $\propto p^2 \frac{\omega_{pe}^2}{\sigma^2}$

The condition of negative permittivity is obtained by:

$$Real(\varepsilon_z) < 0\,; \tag{6}$$

$$\text{If } \omega^2 < \omega_{pe}^2 - \xi_e{}^2; \tag{7}$$

Similarly the Lorentz model of permeability is defined as:

$$\mu_z = 1 - \frac{F\omega^2}{\omega^2 - \omega_{0m}^2 + j\omega\xi_m}$$

$$= 1 - \frac{F\omega^2(\omega^2 - \omega_{0m}^2)}{(\omega^2 - \omega_{0m}^2)^2 + (\omega\xi_m)^2} + j\frac{F\omega^2\xi_m}{(\omega^2 - \omega_{0m}^2)^2 + (\omega\xi_m)^2} \tag{8}$$

$\omega_{0m} \rightarrow$ Magnetic resonance frequency;

$F \rightarrow$ Filling factor;

$\xi_m \rightarrow$ Damping factor due to metal loss $\propto \frac{p}{\sigma}$;

The condition of negative permeability is obtained by:

$$Real(\mu_z) < 0\,; \tag{9}$$

$$\omega_{0m} < \omega < \frac{\omega_{0m}}{\sqrt{1-F}} = \omega_{pm}\,; \tag{10}$$

$\omega_{pm} \rightarrow$ Magnetic plasma frequency (resonant);

Since

$$\beta_z = nk_0 = \pm\sqrt{\varepsilon_z\mu_z} \tag{11}$$

For negative refractive index condition

$$Real(\varepsilon_z), Real(\mu_z) < 0\,; \tag{12}$$

In order to satisfy casualty condition, negative sign is chosen to obtain real part of refractive index:

$$Real(n) < 0; \tag{13}$$

Here we can see that, a simultaneous negative index of permittivity and permeability results in negative index of refraction.

Consider a uniaxial M-NRI medium with anisotropy along z-direction. The optical axis is directed along +ve z-axis. The dispersion relation for TE and TM polarized wave propagation inside 1D M-NRI medium is given as:

TE case:

$$\frac{\beta_z^2}{\mu_T} + \frac{\beta_x^2}{\mu_T} = k_0^2\varepsilon_T \tag{14}$$

TM case:

$$\frac{\beta_z^2}{\varepsilon_T} + \frac{\beta_x^2}{\varepsilon_T} = k_0^2\mu_T \tag{15}$$

$k_0 \rightarrow$ Free space wave number

Transmission and reflection coefficients can be computed on Air-M-NRI interface in terms of propagation vectors in both mediums [4] Fig. 2.

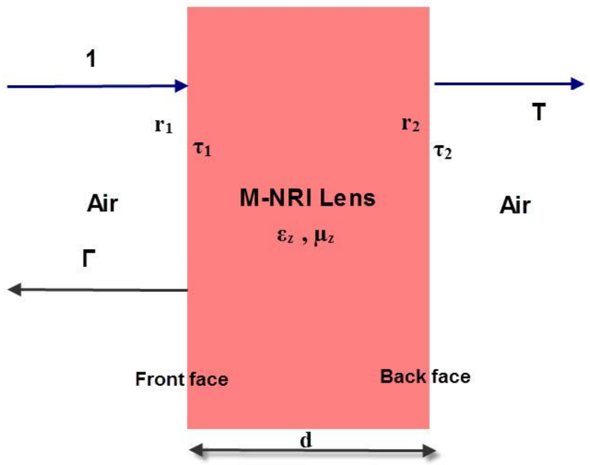

Fig. 2. Reflection and transmission from metamaterial negative refractive index lens (M-NRI Lens).

Consider the TE polarized case; Wave is incident obliquely at free space on M-NRI lens. The transmission coefficient (τ_1) and reflection coefficient (r_1) at this interface are:

$$\tau_1 = \frac{2\mu_T k_z}{\mu_T k_z + \beta_z} \tag{16}$$

$$r_1 = \frac{\mu_T k_z - \beta_z}{\mu_T k_z + \beta_z} \tag{17}$$

$k_z \rightarrow$ Normal component of wave vector in free space
$\beta_z \rightarrow$ Wave vector in NRI medium along +z direction
$d \rightarrow$ Thickness of M-NRI slab;

Similarly, computing the coefficients (τ_2 and r_2) at NRI-free space interface as:

$$\tau_2 = \frac{2\beta_z}{\mu_T k_z + \beta_z} \tag{18}$$

$$r_2 = \frac{\beta_z - \mu_T k_z}{\mu_T k_z + \beta_z} \tag{19}$$

Before final transmission through NRI slab; wave undergoes multiple reflections at both front and back interfaces. We can obtain the transfer function (T) of M-NRI lens by summation of infinite geometric progression as:

$$T = \tau_1\tau_2 \exp(-j\beta_z d) + \tau_1\tau_2\, r_2^2 \exp(-3j\beta_z d) + \tau_1\tau_2\, r_2^4 \exp(-5j\beta_z d) \ldots\ldots \tag{20}$$

$$T = \frac{\tau_1\tau_2 \exp(-j\beta_z d)}{1 - r_2^2 \exp(-2j\beta_z d)}; \tag{21}$$

$$Transmission\ loss = \left|1 - \frac{\tau_1\tau_2 \exp(-j\beta_z d)}{1 - r_2^2 \exp(-2j\beta_z d)}\right| \tag{22}$$

For uniaxial NRI medium, dispersion model of permittivity and permeability can be used to obtain the transmission property of NRI medium. The Transfer function (T) is dispersive and depends upon the thickness of M-NRI medium, conductivity of the metal used, the filling factor & frequency dependent permittivity and permeability of NRI medium.

III. The Q-factor and Losses in Metamaterial

The losses in M-NRI lenses is often described by Quality factor (Q-factor) or figure of merit (FOM) [5]. Higher the Q-factor lower will be the losses. The total Q-factor has contributions from couplings, inhomogeneous broadening & radiation losses. In addition there is an intrinsic Q_0 related to the ohmic damping of surface plasmons. The different Q-factors can be related as:

$$\frac{1}{Q} = \frac{1}{Q_0} + \frac{1}{Q_{coup}} + \frac{1}{Q_{rad}} + \frac{1}{Q_{inhom}} \tag{22}$$

Any structure made of resonating elements, in general, does not constitute good transmission medium for a modulated signals due to the Q-factor associated with each resonator [3]. The loaded Q-factor of the structure is given as:

$$\frac{1}{Q_{loaded}} = \frac{1}{Q_{unloaded}} + \frac{1}{Q_{external}} \tag{23}$$

Transmission loss ($1/Q_{loaded}$) through any resonator is the sum of dielectric/ohmic loss ($1/Q_{unloaded}$) in the resonator and the coupling/mismatch losses ($1/Q_{external}$) in transition with external load.

Consider a semi-infinite M-NRI medium with permittivity ($\varepsilon=\varepsilon_0\varepsilon_r$) and permeability ($\mu=\mu_0\mu_r$). The relative wave impedance of M-NRI lens medium is defined as:

$$\eta_r = \sqrt{\frac{\mu_r}{\varepsilon_r}} \tag{24}$$

At the interface between the slab and free-space, the reflection coefficient for normal incidence has the well-known form.

$$\Gamma = \frac{\sqrt{\frac{\mu_r}{\varepsilon_r}} - 1}{\sqrt{\frac{\mu_r}{\varepsilon_r}} + 1} = \frac{\eta_r - 1}{\eta_r + 1} \tag{25}$$

For a slab of finite thickness (d) of M-NRI medium, the reflection and transmission coefficients for plane wave scattering at normal incidence are given as S_{11} & S_{21} respectively. For normal incidence from air to air-MNRI-Lens interface, Reflection and transmission S-parameters are in general related to relative wave impedance as:

$$S_{11} = \frac{(\eta_r^2 - 1)(1 - Z^2)}{(\eta_r + 1)^2 - (\eta_r - 1)^2 Z^2} \tag{26}$$

$$S_{21} = \frac{4\eta_r Z}{(\eta_r + 1)^2 - (\eta_r - 1)^2 Z^2} \tag{27}$$

Where the transmission term:

$$Z = \exp(-jkd) \tag{28}$$

If the slab is to be matched to free space i.e.$\eta_r = 1$, then

$$S_{11} = 0 \tag{29}$$

$$S_{21} = Z \tag{30}$$

Hence the frequency band in which the matched transmission occurs is identified by:

$$|(S_{11})| = 0 \text{ and } |(S_{21})| = 1 \tag{31}$$

Q_{loaded} is the quantity which can be actually measured and relevant in terms of transmission and can be obtained in terms of magnitude of (S_{21}).

$$\frac{1}{Q_{loaded}} = \frac{B}{f_r} \tag{32}$$

Where B is -3dB bandwidth of (S_{21}) and f_r is resonant frequency. It is clear from Eqn. 32, that higher the transmission bandwidth (B) losses will be more.

From Equation 22 and Equation 27, the relationship between transmission loss and S-parameters can be roughly estimated as:

$$Transmission\ loss = \left| 1 - \frac{\tau_1 \tau_2 \exp(-j\beta_z d)}{1 - r_2^2 \exp(-2j\beta_z d)} \right| \approx 1 - K. \left| (S_{21}) \right| \qquad (33)$$

K → Proportionality Constant

IV. Loss Compensation Schemes of Metamaterial Negative Refractive Index Lens

Losses are the main obstacle in practical realization of M-NRI lens. Despite the rapid progress on metamaterials, experimental M-NRI lens realizations are very few. It is thus essential to find a way out to reduce the losses in this lens. The most obvious approach is to introduce gain in the M-NRI lens medium. Different techniques of loss compensation have been proposed in literature. Some important loss compensation techniques are:

a) Introduction of gain in the medium:

- Embedding active circuits: Negative resistance region of nonlinear diode can be used to mitigate losses in NRI medium. The major drawback of this technique is its practical realization. Biasing is also required that makes the system more impractical.[6]

- Pump process: This is optical pumping based method for loss compensation. It may be highly impractical and unscalable. This process leads to the inversion of the gain system necessary for compensation of the metal losses at the probe wavelength. During the short duration of pumping, a strong nonlinear response can be observed in the occupation densities. In this process the pump pulse produces a long-lived population inversion.[7]

b) Electrical approach: Injection of electrical current from semiconductor into chunk of M-NRI medium e.g. inserting additional SRR smaller in size with amplifier signal chain to sense, amplify and inject amplified signal into metamaterial to compensate for loss. Main drawback in using this technique is an additional antenna element which may offset the original M-NRI response.[8][9]

c) Geometrical approach: Suggests to void corners & sharp edges in designing M-NRI lenses.[10]

d) Use of noble conductors: The losses in NRI materials originate from intrinsic absorption of constituent materials, specifically metals which are highly lossy at optical frequencies. Resonant nature and surface roughness also contributes to the losses. These losses severely hinder the performance of metamaterials and restrict their range of practical applications. Noble conductors such as silver and gold should be used to reduce losses in the M-NRI medium.[11]

e) Optical parametric amplification: In this process, the wave vectors of all three coupled waves are codirected, whereas the energy flow for the signal negative-index wave is counter directed with respect to others for the pump and the idler waves. Here a positive-index control field enables a loss-balancing OPA for a negative index signal wave.[12]

f) Avoiding use of Metals: Metals generally offer negative real permittivity. They are accompanied by large losses. Even noble metals like silver and gold, with the highest DC conductivities exhibit excessive losses at optical frequencies that seriously restrict the experimental realization of such novel M-NRI devices. Thus nonmetallic designs of NRI materials can achieve much less attenuation than metallic designs.[13]

V. Conclusion

In conclusion, we have discussed & showed the feasibility of compensating losses in NRI medium by various loss compensation techniques. A small amount of loss can dramatically modify the expected response of MNRI lens to some degree, but does not destroy the collimating effect of the lens. However the techniques discussed above for loss compensation in NRI medium are more theoretical and not much experimental validation results are available for the moment. Still these techniques provide guidelines in designing low loss NRI lenses and can be combined to achieve loss compensation over a broad frequency range

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