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IMPROVEMENT OF SUPPLY CHAIN MANAGEMENT BY MATHEMATICAL PROGRAMMING APPROACH

R.K.VERMA[1], K.M.MOEED[2], K.G.SINHA[3], KAPIL CHOPRA[4]
[1]Research scholar, Integral University
[2]Associate Professor, Integral University
[3]Assistant Professor SR Institute of Management and Technology
[4]Assistant Professor BNCET, Lucknow
Department of Mechanical Engineering

Abstract**—**this paper analyses the case of any production system by mathematical programming approach of a model for the existing or a new industry we can analyze the different aspects of manufacturing costs and then by using various techniques we can minimize the total cost from one end to another end so that the manufacturing cost decreases and profit increases.

Index Terms**—**supply chain management, productivity, mathematical programming approach. *(key words)*

I. INTRODUCTION

Mathematical programming is one of the best tool available for quantitative decision making. The general purpose of mathematical programming is to find out an optimal solution for available allocation of limited resources to perform competing activities. The optimality may be defined with respect to important performance evaluation criteria, such as cost, time, and profit. Mathematical programming uses a compact mathematical model for describing the problem of concern. The solution is searched among all feasible alternatives for finding out optimal solution. The search is executed in an intelligent manner, allowing the evaluation of problems with a large number of feasible solutions for decision making.

Mathematical programming finds many applications in supply chain management, at all decision-making levels. It is also widely used for supply chain configuration purposes. Out of several classes of mathematical programming models, mixed-integer programming models are used most frequently. Other types of models, such as stochastic and multi-objective programming models, are also emerging to handle more complex supply chain configuration problems. Although these models are often more appropriate, computational complexity remains an important issue in the application of mathematical programming models for supply chain configuration.

This investigation is aimed to describe application of mathematical programming for supply chain configuration. It is followed by a description of generic supply chain configuration mixed integer programming model. Computational approaches for solving problems of large size are also discussed along with typical modifications of the generic model, especially, concerning global factors.

II. FUNDAMENTALS

Mathematical programming models are used to optimize decisions concerning execution of certain activities subject to resource constraints. Mathematical programming models have a well-defined structure. They consist of mathematical expressions representing objective function and constraints. The expressions involve parameters and decision variables. The parameters are input data, while the decision variables represent the optimization outcome. The objective function represents modelling objectives and makes some decisions more preferable than others. The constraints limit the values that decision variables can assume.

The main advantages of mathematical programming models are that they provide a relatively simple and compact approximation of complex decision-making problems, an ability to efficiently find an optimal set of decisions among a large number of alternatives, and supporting analysis of decisions made. Specifically, in the supply chain configuration problem context, mathematical programming models are excellent for modelling its special aspects.

There are also some important limitations. Mathematical programming models have a lower level of validity compared to some other types of models particularly, simulation. In the supply chain configuration context, mathematical programming models have difficulties representing the dynamic and stochastic aspects of the problem. Additionally, solving of many supply chain configuration problems is computationally challenging.

Following the supply chain configuration scope, mathematical programming models are suited to answer the following supply chain configuration questions:

1. Which partners to choose?
2. Where to locate supply chain facilities?
3. How to allocate production and capacity?
4. Which transportation mode to choose?
5. How do specific parameters influence supply chain performance?

The most common type of mathematical programming models is linear programming models. These models have all constraints and the objective function expressed as a linear function in variables. However, many real- life problems cannot be represented as linear functions. A typical example is representation of decisions concerning the opening of supply chain facilities. These decisions assume values equal either to 0 or 1. Integer programming models are used to model such problems. Their computational tractability is lower than that of linear programming models. Non-linear expressions are often required to represent inventory and transportation-related issues of supply chain. That results in nonlinear programming models, which have high computational complexity.

Given the heterogeneous nature of supply chains, optimization often cannot be performed with respect to a single objective. Multi-objective programming models seek an optimal solution with regard to multiple objectives. These models rely on judgmental assessment of the relative importance of each objective.

Generally, as one moves from linear programming to more complex mathematical programming models, the validity of representing real-world problems is improved at the expense of model development and solving simplicity. Specialized model-solving algorithms are often required to solve complex problems.

Mathematical programming modelling systems (Greenberg 1993)[1] have been developed for elaboration, solving, and analysis of mathematical programming models. These include GAMS, ILOG, and LINGO, to mention a few. These systems provide a means for data handling, model composition using special-purpose mathematical programming languages, and model solving. From the perspective of integrated decision modelling frameworks, these systems can be easily integrated into the decision support system to provide optimization functionality. The integration is achieved by using some types of application programming interfaces. Data structures used, generally, are system specific. Therefore, these need to be mapped to data sources using information modelling.

The role of mathematical programming systems in the overall strategic decision-making system has been described by Shapiro (2000)[2]. The described optimization modelling system includes links from the mathematical programming system to a decision-making database and other data sources, as well as advanced tools for conducting analysis. Generation of optimization models from data stored in the decision-making database is considered.

III. MIXED-INTEGER PROGRAMMING MODELS

Traditional supply chain configuration models are mixed integer programming models. This section starts with presenting a generic model formulation which includes only the most frequently used decision variables, parameters, and constraints, as identified during construction of the generic supply chain configuration data model. The presentation of the generic model is followed by an overview of most frequently used modifications.

A. GENERIC FORMULATION

The following sub-sections define notation used to specify the generic supply chain configuration optimization model, and present the object function and constraints of this model.

Notations	Definition
I	products
J	materials
K	plants
S	suppliers
M	distribution centres
N	customers
parameters	
d_{in}	demand
h_k	plant capacity
γ_i	capacity requirements for products
δ_{ij}	materials consumption per products
ω_{js}	material purchasing cost from supplier per unit
λ_{ik}	production cost at plant per unit
r_{im}	handling cost at distribution centre per unit
t_{1jsk}	transportation cost from supplier to plant per material unit
t_{2ikm}	transportation cost from plant to distributer per product unit
t_{3mn}	transportation cost from distribution centre to customer per product unit
f_{1k}	plant fixed opening/operating cost
f_{2k}	distribution centres fixed cost
P	Constant
Decision variables	
X_{imn}	Quantity of production sold from distribution centre to customer
Q_{ik}	Quantity of products produced at plant
Y_{ikm}	Quantity of product shipped from plant to distribution centre
V_{jsk}	Quantity of material purchased and shipped from supplier to plant
W_k	Plant open indicator equals 1 if plant is open and 0 otherwise
U_m	Distribution center open indicator equals 1 if distribution center is open 0 otherwise

B. OBJECTIVE FUNCTION

The objective function minimizes the total cost (TC). As indicated in the previous chapter, minimization of the total cost is considered more often than profit maximization. The total cost consists of production cost, materials purchasing and transportation cost, products transportation cost from plants to distribution centres, product handling and transportation cost from distribution centres to customers, and fixed costs for opening and operating plants and distribution centres.

$$TC = \sum_{i=1}^{I}\sum_{k=1}^{K} O_{ik}\, Q_{ik} + \sum_{j\forall=1}^{J}\sum_{s=1}^{S}\sum_{k=1}^{K}(\omega_{js} + t_{1jsk})V_{jsk} + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{m=1}^{M} t_{2ikm}Y_{ikm} + \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{n=1}^{N}(r_{im} + t_{3imn})X_{imn} + \sum_{k=1}^{K} f_{1k}W_k + \sum_{m=1}^{M} f_{1m}W_m$$

Equation 1

C. CONSTRAINTS

Equation-2

$$\sum_{m=1}^{M} X_{imn} \leq d_i, \forall i, n$$

Equation-3

$$\sum_{n=1}^{N} X_{imn} \leq \sum_{k=1}^{K} Y_{ikm}, \forall i, k$$

Equation-4

$$\sum_{m=1}^{M} Y_{ikim} \leq Q_{ik}, \forall i, k$$

Equation-5

$$\sum_{i=1}^{I} \gamma_i Q_{ik} \leq h_k W_k, \forall k$$

Equation-6

$$\sum_{i=1}^{I} \delta_{ij} Q_{ij} \leq \sum_{s=1}^{S} V_{jsk}, \forall j, k$$

Equation-7

$$\sum_{i-1}^{I} \sum_{n=1}^{N} X_{imn} < PU_m, \forall m$$

Equation-8

$$W_k, U_m \in, \{0,1\} \forall k, m$$

Equation (2) enforces the balance between products sold and demand. The balance between incoming and outgoing flows at distribution centres is defined by Equation (3). The balance between products produced and products shipped to distribution centres is enforced by Equation (4). Equation (5) restricts capacity availability. Availability of materials to produce products is checked by Equation (6) and Equation (7) states that product flows are allowed only through open distribution centres.

D. COMMENTS

The model does not explicitly include parameters characterizing a spatial location of supply chain units. Alternative locations for a particular supply chain unit are evaluated by allowing for several units with equal characteristics but different transportation costs, which characterize the location of the unit.

There are two factors affecting the model composition:

1) The broker and power structure

2) The initial state of the network.

Depending upon the organizational and power structure of the supply chain and a decision maker's point of view (i.e., interests of the whole supply chain vs. interests of the dominant member), some of the cost parameters are set to zero because the total cost the broker is concerned about is not affected by these cost parameters, even if these are relevant to the overall supply chain modelling (e.g., a final assembler pays only purchasing costs for components and is not concerned about processing costs at the supply level). The initial state of the network determines whether some of the decision variables already do not have a fixed value. For instance, the location of several assembly plants is already fixed and cannot be changed. Similarly, long-term purchasing contracts with some suppliers can set definite limits on purchasing volume from these suppliers.

E. MODIFICATIONS

The generic formulation obviously needs to be adjusted to include factors relevant to a particular decision-making problem. Analysis suggests that the most frequently considered factors are international factors, inventory, capacity treatment, transportation, and supply chain management policies.

1) International Factors

Given that many supply chains involve partners from different countries, international factors need to be addressed in supply chain configuration. This problem is of particular importance for large multi-national companies manufacturing and selling their products world-wide. Mathematical programming models consider quantitative factors, while there are also numerous qualitative factors influencing international decision making. Goetschalckx et al. (2002)[3] provide a summary table on works considering international factors. This summary indicates that taxes and duties are the most often considered international factors. In a similar work by Meixell and Gargeya (2005)[4], the most frequently considered international factors besides tariffs and duties are currency exchange rates and corporate income taxes. However, many of the models surveyed use already fixed supply chain configuration. Kouvelis et al. (2004)[5] presents an extensive sensitivity analysis of the impact of international factors on supply chain configuration.

2) Capacity Treatment

A majority of models have some sort of flow intensity and transformation capacity limits as a parameter. A parameter characterizing capacity consumption per unit processed or handled is also widely used. Sabri and Beamon (2000)[6] and Yan et al. (2003) use product specific capacity, while Pirkul and Jayaraman (1998)[7] the flexible capacity. Bhutta et al. (2003)[8] is one of the few papers using capacity as a decision variable. This paper allows either increasing or decreasing capacity at the facility.

3) Transportation

The most common way of representing transportation is considering just one mode and including variable costs per unit shipped between supply chain units. However, transportation-related issues generally are much more complex and several models attempt to account for this complexity. Non-linear dependence of transportation costs according to quantity shipped is modelled by Tsiakis et al. (2001)[9]. This dependence is represented by a piece-wise linear function. Transportation costs are not calculated for individual products but for families of similar products, thus reducing the model complexity. Syam (2002)[10] include a fixed charge per unit using a particular link to transfer products between units. Ross et al. (1998)[11] has transportation as one of the key specific problems of supply chain configuration decision making and the model represents individual vehicles with their characteristics. Arntzen et al. (1995) and Syam (2002[12] represent transportation capacity by limiting the total weight of products shipped. The shipment weight-based representation of shipments costs and transportation capacity is often used in applied studies.

Detailed representation of transportation is a feature of many commercial supply chain network design models. These are based on detailed databases of distance and freight rates. These data as well as transportation cost structure and shipment planning are described by Bowersox et al. (2002)[13].

IV. CASE STUDY

A. SCOOTERS INDIA LIMITED

1) GENERIC FORMULATION

Notations	Definition		
I	products	Gear 1	Gear 2
j	materials	EN 36(Rs500/Kg)	EN 45(Rs450/kg)
K	plants	8	8
s	suppliers	3	3
m	distribution centres	0	0
n	customers	0	0
parameters			
d_{in}	demand	300	300
h_k	plant capacity	500	500
γ_i	capacity requirements for products	350	350
δ_{ij}	materials consumption per products	400gm	400gm
ω_{js}	material purchasing cost from supplier per unit	32	30
λ_{ik}	production cost at plant per unit	20	20
r_{im}	handling cost at distribution centre per unit	2	2
t_{1jsk}	transportation cost from supplier to plant per material unit	3	3
t_{2ikm}	transportation cost from plant to distributer per product unit	1	1
t_{3mn}	transportation cost from distribution centre to customer per product unit	5	5
f_{1k}	plant fixed opening/operating cost	2	2
f_{2k}	distribution centres fixed cost	3	3
P	constant	10	10
Decision variables			
X_{imn}	Quantity of production sold from distribution centre to customer	250	250
Q_{ik}	Quantity of products produced at plant	1200	1200
Y_{ikm}	Quantity of product shipped from plant to distribution centre	500	500
V_{jsk}	Quantity of material purchased and shipped from supplier to plant	450	450
W_k	Plant open indicator equals 1 if plant is open and 0 otherwise	1	1
U_m	Distribution center open indicator equals 1 if distribution center is open otherwise 0	1	1

$$TC = \sum_{i=1}^{I}\sum_{k=1}^{K} O_{ik}\, Q_{ik} + \sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{k=1}^{K}(\omega_{js} + t_{1jsk})V_{jsk} + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{m=1}^{M} t_{2ikm}Y_{ikm} + \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{n=1}^{N}(r_{im} + t_{3imn})X_{imn} + \sum_{k=1}^{K} f_{1k}W_k + \sum_{m=1}^{M} f_{1m}W_m$$

$$TC = \sum_{i=1}^{1}\sum_{k=1}^{8} 20\frac{1}{1200} + \sum_{j=1}^{500}\sum_{s=1}^{3}\sum_{k=1}^{8}(32+3)\frac{1}{450} + \sum_{i=1}^{1}\sum_{k=1}^{8}\sum_{m=1}^{0} 1*\frac{1}{500} + \sum_{i=1}^{1}\sum_{m=1}^{0}\sum_{n=1}^{0}(2+5)\frac{1}{250} + \sum_{k=1}^{8} 2*1 + \sum_{m=1}^{0} 3*1$$

TC = (0.33+933+0.16+7+0)

TC = Rs.940.49/gear

$$TC = \sum_{i=1}^{1}\sum_{k=1}^{8} 20\frac{1}{1200} + \sum_{j=1}^{450}\sum_{s=1}^{3}\sum_{k=1}^{8}(30+3)\frac{1}{450} + \sum_{i=1}^{1}\sum_{k=1}^{8}\sum_{m=1}^{0} 1*\frac{1}{500} + \sum_{i=1}^{1}\sum_{m=1}^{0}\sum_{n=1}^{0}(2+5)\frac{1}{250} + \sum_{k=1}^{8} 2*1 + \sum_{m=1}^{0} 3*1$$

TC = (0.33+792+0.16+7+0)

TC = Rs.799.49/gear

B. OMAX AUTOS LIMITED

1) GENERIC FORMULATION

Notations	Definition		
I	products	2782310	2782358
j	materials	E-46 SS 4012A	E-53 SS 5015B
K	plants	1	1
s	suppliers	1	1
m	distribution centres	1	1
n	customers	1	1
parameters			
d_{in}	Demand	250	350
h_k	plant capacity	300	400
γ_i	capacity requirements for products	300	400
δ_{ij}	materials consumption per products	$0.5m^2$	$0.6m^2$
ω_{js}	material purchasing cost from supplier per unit	Rs.79/m^2	Rs.85/m^2
λ_{ik}	production cost at plant per unit	25	25
r_{im}	handling cost at distribution centre per unit	5	5
t_{1jsk}	transportation cost from supplier to plant per material unit	3	3
t_{2ikm}	transportation cost from plant to distributer per product unit	1	1
t_{3mn}	transportation cost from distribution centre to customer per product unit	5	5
f_{1k}	plant fixed opening/operating cost	2	2
f_{2k}	distribution centres fixed cost	3	3
P	Constant	10	10
Decision variables			
X_{imn}	Quantity of production sold from distribution centre to customer	300	300
Q_{ik}	Quantity of products produced at plant	250	250
Y_{ikm}	Quantity of product shipped from plant to distribution centre	200	200
V_{jsk}	Quantity of material purchased and shipped from supplier to plant	200	200
W_k	Plant open indicator equals 1 if plant is open and 0 otherwise	1	1
U_m	Distribution center open indicator equals 1 if distribution center is open otherwise 0	1	1

$$TC = \sum_{i=1}^{I}\sum_{k=1}^{K} O_{ik}\, Q_{ik} + \sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{k=1}^{K}(\omega_{js} + t_{1jsk})V_{jsk}$$
$$+ \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{m=1}^{M} t_{2ikm} Y_{ikm}$$
$$+ \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{n=1}^{N}(r_{im} + t_{3imn})X_{imn} + \sum_{k=1}^{K} f_{1k}W_k + \sum_{m=1}^{M} f_{1m}W_m$$

$$TC = \sum_{i=1}^{1}\sum_{k=1}^{1} 79\frac{1}{250} + \sum_{j=1}^{1}\sum_{s=1}^{1}\sum_{k=1}^{1}(79+3)\frac{1}{200}$$
$$+ \sum_{i=1}^{1}\sum_{k=1}^{1}\sum_{m=1}^{1} 1 * \frac{1}{250}$$
$$+ \sum_{i=1}^{1}\sum_{m=1}^{0}\sum_{n=1}^{0}(3+5)\frac{1}{300} + \sum_{k=1}^{1} 3 * 79 + \sum_{m=1}^{0} 2 * 1$$

TC = (0.316+.405+.004+.032+237+2)

TC = Rs.339.757/product

$$TC = \sum_{i=1}^{1}\sum_{k=1}^{1} 85\frac{1}{350} + \sum_{j=1}^{1}\sum_{s=1}^{1}\sum_{k=1}^{1}(85+3)\frac{1}{200} + \sum_{i=1}^{1}\sum_{k=1}^{1}\sum_{m=1}^{1} 1*\frac{1}{350} + \sum_{i=1}^{1}\sum_{m=1}^{0}\sum_{n=1}^{0}(3+5)\frac{1}{300} + \sum_{k=1}^{1} 3*85 + \sum_{m=1}^{0} 2*1$$

TC= (0.24+.440.002857+0.026+255+2))

TC = Rs.257.7094/product

V. Conclusion

The investigation can be concluded in following words

- We can compare the cost between two or three products.
- We can optimise the production in terms of cost as well as in terms of production system.
- We can forecast the cost of finished good at the starting of production.

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