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τ^* Generalized Preclosed Sets In Topological Spaces

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Abstract— **In this paper, we introduce a new class of sets called -generalized preclosed sets and -generalized preopen sets in topological spaces and study some of their properties.**

Index terms- $\tau^* - gp - closed$ set, $\tau^* - gp - open$ set

I. Introduction

In 1970, Levine[6] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Using generalized closed sets, Dunham[5] introduced the concept of the closure operator c1* and a new topology τ^* and studied some of their properties. S.P.Arya[2], P.Bhattacharyya and B.K.Lahiri[3], J.Dontchev[4], H.Maki, R.Devi and K.Balachandran[9], [10], P.Sundaram and A.Pushpalatha[12], A.S.Mashhour, M.E.Abd E1-Monsof and S.N.E1-Deeb[11], D.Andrijevic[1] and S.M.Maheshwari and P.C.Jain[9], Ivan Reilly [13], A.Pushpalatha, S.Eswaran and P.RajaRubi [14] introduced and investigated generalized semi closed sets, semi generalized closed sets, generalized semi preclosed sets, $\alpha -$generalized closed sets, generalized-α closed sets, strongly generalized closed sets, preclosed sets, semi-preclosed sets and $\alpha -$closed sets, generalized preclosed sets and τ^*-generalized closed sets respectively. In this paper, we obtain a new generalization of preclosed sets in the weaker topological space(X, τ^*).

Throughout this paper X and Y are topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A of a topological space X, int(A), $cl(A)$, cl^*(A), $scl(A), spcl(A), cl_\alpha(A)$, $cl_p(A)$ and A^c denote the interior, closure, $closure^*$, semi-closure, semi-preclosure, α-closure, preclosure and complement of A respectively.

2.Preliminaries:

We recall the following definitions

Definition: 2.1

A subset A of a topological space (X , τ) is called

(i) Generalized closed (briefly g-closed)[6] if cl(A)⊆G whenever A⊆G and G is open in X

(ii) Semi-generalized closed(briefly sg-closed)[3] if scl(A)⊆G whenever A⊆G and G is semi open in X.

(iii) Generalized semi-closed (briefly gs-closed)[2] if scl(A)⊆G whenever A⊆G and G is open in X.

(iv) α- closed[8] if cl(int(cl(A))) ⊆ A

(v) α-generalized closed (briefly αg-closed)[9] if cl_α(A) ⊆ G whenever A⊆G and G is open in X.

(vi) Generalized α-closed (briefly gα-closed)[10] if spcl(A) ⊆ G whenever A⊆G and G is open in X.

(vii) Generalized semi-preclosed (briefly gsp-closed)[2] if scl(A) ⊆ G whenever A ⊆ G and G is open in X.

(viii) Strongly generalized closed (briefly strongly g-closed)[12] if $cl(A) \subseteq G$ whenever $A \subseteq G$ and G is g-open in X.

(ix) Preclosed[11] if $cl(int(A)) \subseteq A$

(x) Semi-closed[7] if $int(cl(A)) \subseteq A$

(xi) Semi-preclosed (briefly sp-closed)[1] if $int(cl(int(A))) \subseteq A$.

(xii) Generalized preclosed (briefly gp-closed)[13] if $cl_p A \subseteq G$ whenever $A \subseteq G$ and G is open.

The complements of the above mentioned sets are called their respective open sets.

Definition: 2.2

For the subset A of a topological X, the generalized closure operator cl^*[5] is defined by the intersection of all g-closed sets containing A.

Definition: 2.3

For the subset A of a topological X, the topology τ^* is defined by $\tau^* = \{G: cl^*(G^c) = G^c\}$.

Definition: 2.4

For the subset A of a topological X,

(i) the semi-closure of A$(briefly\ scl(A))$[7] is defined as the intersection of all semi-closed sets containing A.

(ii) the semi-Pre closure of A$\left(briefly\, spcl(A)\right)$[1] is defined as the intersection of all semi-preclosed sets containing A.

(iii) the $\alpha - closure$ of A $\left(briefly\, cl_\alpha(A)\right)$[8] is defined as the intersection of all $\alpha - closed$ sets containing A.

(iv) the preclosure of A, denoted by $cl_p(A)$[13], is the smallest preclosed set containing A.

Definition: 2.5

A subset A of a topological space X is called τ^* generalized closed set $(briefly\, \tau^* - gclosed)$[14] if $cl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^* - open$. The complement of $\tau^* - generalized$ closed set is called the $\tau^* - generalized$ open set $(briefly\, \tau^* - g - open)$.

3. $\tau^* - Generalized$ Preclosed Sets In Topological Spaces

In this section, we introduce the concept of τ^*- generalized preclosed sets in topological spaces.

Definition: 3.1

A subset A of a topological space X is called $\tau^* - generalized$ preclosed $(briefly\, \tau^* - gp - closed)$ if $cl^*(cl_p(A)) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^* -$ open. The complement of $\tau^* - generalized$ preclosed set is called the $\tau^* - generalized$ preopen set (briefly $\tau^* - gp - open$).

Example: 3.2

Let $X = \{a, b, c\}$ and let $\tau = \{\phi, X, \{a\}, \{a, b\}, \{c\}, \{a, c\}\}$. Here (X, τ^*) is τ^*-generalized preclosed

Theorem: 3.3

Every closed set in X is $\tau^* - gp - closed.$

Proof:

Let A be a closed set. Let $A \subseteq G$. Since A is closed, $cl(A) = A \subseteq G$. But $cl^*(cl_p(A)) \subseteq cl(A)$. Thus, we have $cl^*(cl_p(A)) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^* - open$. Therefore A is $\tau^* - gp - closed.$

Theorem: 3.4

Every $\tau^* - closed$ set in X is $\tau^* - gp - closed.$

Proof:

Let A be a $\tau^* - closed$ set. Let $A \subseteq G$ where G is $\tau^* - open$ Since A is $\tau^* - closed$, $cl^*(A) = A \subseteq G$. Thus, we have $cl^*\left(cl_p(A)\right) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^* - open$. Therefore A is $\tau^* - gp$ closed.

Theorem: 3.5

Every g-closed set in X is a τ^*–gp-closed set but not conversely.

Proof:

Let A be a g-closed set. Assume that $A \subseteq G$, G is $\tau^* - open$ in X. Then $cl(A) \subseteq G$, Since A is g-closed. But $cl^*\left(cl_p(A)\right) \subseteq cl(A)$. Therefore $cl^*(cl_p(A)) \subseteq G$. Hence A is $\tau^* - gp - closed.$

The converse of the above theorem need not be true as seen from the following example.

Example: 3.6

Consider the topological space $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{a, b\}, \{c\}, \{a, c\}\}$. Then, the set {a,c} is $\tau^* - gp$-closed but not g-closed.

Remark: 3.7

The following example shows that $\tau^* - gp - closed$ sets are independent from sp-closed, sg-closed set, $\alpha - closed\, set$, preclosed set, gs-closed set, gsp-closed set, $\alpha g - closed$ set and g$\alpha - closed$ set.

Example: 3.8

Let $X = \{a, b, c\}$ and $Y = \{a, b, c, d\}$ be the topological spaces.

(i) Consider the topology $\tau = \{X, \phi, \{a\}\}$. Then the sets $\{a\}, \{a, b\}\, and\, \{a, c\}$ are $\tau^* - gp - closed$ but not sp-closed.

(ii) Consider the topology $\tau = \{X, \phi, \{a, b\}\}$. Then the sets $\{a\}$ and $\{b\}$ are sp-closed but not $\tau^* - gp - closed.$

(iii) Consider the topology $\tau = \{X, \phi\}$. Then the sets $\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\, and\, \{a, c\}$ are $\tau^* - gp - closed$ but not sg-closed.

(iv) Consider the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then the sets $\{a\}\, and\, \{b\}$ are sg-closed but not $\tau^* - gp - closed.$

(v) Consider the topology $\tau = \{X, \phi, \{a\}\}$. Then the sets $\{a\}, \{b\}, \{c\}, \{a, b\}\, and\, \{a, c\}$ are $\tau^* - gp - closed$ but not $\alpha - closed.$

(vi) Consider the topology $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. Then the set $\{b\}$ is $\alpha - closed$ but not $\tau^* - gp - closed.$

(vii) Consider the topology $\tau = \{X, \phi, \{a\}\}$. Then the sets $\{a\}\, \{a, b\}\, and\, \{a, c\}$ are $\tau^* - gp - closed$ but not preclosed.

(viii) Consider the topology $\tau = \{X, \phi, \{b\}, \{a, b\}\}$. Then the set $\{a\}$ is pre-closed but not $\tau^* - gp - closed$.

(ix) Consider the topology $\tau = \{X, \phi\}$. Then the sets $\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}$ and $\{a, c\}$ are $\tau^* - gp - closed$ but not gs-closed.

(x) Consider the topology $\tau = \{Y, \phi, \{a\}, \{a, b, c\}, \{a, b, d\}\}$.Then the sets $\{b\}$ $\{b, c\}$ and $\{b, d\}$ are $gs - closed$ but not $\tau^* - gp -$closed.

(xi) Consider the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then the sets $\{b\}$ and $\{a, b\}$ are $gsp - closed$ but not $\tau^* - gp -$closed.

(xii) Consider the topology $\tau = \{Y, \phi, \{a\}\}$. Then the set $\{a\}$ is $\tau^* - gp - closed$ but not gsp- closed.

(xiii) Consider the topology $\tau = \{X, \phi, \{a\}\}$. Then the set $\{a\}$ is $\tau^* - gp - closed$ but not αg-closed.

(xiv)Consider the topology $\tau = \{Y, \phi, \{a\}, \{a, b, c\}, \{a, b, d\}\}$. Then the set $\{b\}, \{b, c\}, \{b, d\}$ are $\alpha g - closed$ but not $\tau^* - gp - closed$.

(xv) Consider the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Then the set $\{b\}$ is $\tau^* - gp - closed$ but not g$\alpha - closed$.

(xvi)Consider the topology $\tau = \{Y, \phi, \{a\}, \{a, b, c\}, \{a, b, d\}\}$. Then the set $\{b\}, \{b, c\}$ and $\{b, d\}$ are $g\alpha - closed$ but not $\tau^* - gp - closed$.

Theorem: 3.9

For any two sets A and B

$$cl^*\left(cl_p(A \cup B)\right) = cl^*\left(cl_p(A)\right) \cup cl^*\left(cl_p(B)\right)$$

Proof:

Since A$\subseteq A \cup B$, we have $cl^*\left(cl_p(A)\right) \subseteq cl^*\left(cl_p(A \cup B)\right)$ and Since $B \subseteq A \cup B$, we have $cl^*\left(cl_p(B)\right) \subseteq cl^*\left(cl_p(A \cup B)\right)$. Therefore $cl^*\left(cl_p(A)\right) \cup cl^*\left(cl_p(B)\right) \subseteq cl^*\left(cl_p(A \cup B)\right)$. Also, $cl^*\left(cl_p(A)\right)$ and $cl^*\left(cl_p(B)\right)$ are the closed sets.

Therefore $cl^*\left(cl_p(A)\right) \cup cl^*\left(cl_p(B)\right)$ is also a closed set. Again, A$\subseteq cl^*\left(cl_p(A)\right)$ and B$\subseteq cl^*\left(cl_p(B)\right)$, Implies $\cup B \subseteq cl^*\left(cl_p(A)\right) \cup cl^*\left(cl_p(B)\right)$. Thus, $cl^*\left(cl_p(A)\right) \cup cl^*\left(cl_p(B)\right)$ is a closed set containing $A \cup B$. Since $cl^*\left(cl_p(A \cup B)\right)$ is the smallest closed set containing $A \cup B$. We have

$cl^*\left(cl_p(A \cup B)\right) \subseteq cl^*\left(cl_p(A)\right) \cup cl^*\left(cl_p(B)\right)$.

Thus, $cl^*\left(cl_p(A \cup B)\right) = cl^*\left(cl_p(A)\right) \cup cl^*\left(cl_p(B)\right)$

Theorem: 3.10

Union of two $\tau^* - gp - closed$ sets in X is a $\tau^* - gp - closed\, set\, in\, X$.

Proof:

Let A and B be two $\tau^* - gp - closed\, sets$. Let $A \cup B \subseteq G$, where G is $\tau^* - open$. Since A and B are $\tau^* - gp - closed\, sets$, $cl^*\left(cl_p(A)\right) \cup cl^*\left(cl_p(B)\right) \subseteq G$. But by theorem 3.12 $cl^*\left(cl_p(A)\right) \cup cl^*\left(cl_p(B)\right) = cl^*\left(cl_p(A \cup B)\right)$. Ther efore $cl^*\left(cl_p(A \cup B)\right) \subseteq G$. Hence $A \cup B$ is a $\tau^* - gp - closed$ set.

Theorem: 3.11

A subset A of X is $\tau^* - gp - closed$ if and only if $cl^*\left(cl_p(A)\right) - A$ contains no non-empty $\tau^* - closed$ set in X.

Proof:

Let A be a $\tau^* - gp - closed$ set. Suppose that F is a non-empty $\tau^* - closed$ subset of $cl^*\left(cl_p(A)\right) - A$. Now, F$\subseteq cl^*\left(cl_p(A)\right) - A$. Then F$\subseteq cl^*\left(cl_p(A)\right) \cap A^c$ Since $cl^*\left(cl_p(A)\right) - A = cl^*\left(cl_p(A)\right) \cap A^c$. Therefore F$\subseteq cl^*\left(cl_p(A)\right)$ and F$\subseteq A^c$. Since F^c is a τ^*-open set and A is a $\tau^* - gp - closed$, $cl^*\left(cl_p(A)\right) \subseteq F^c$. That is $F \subseteq cl^*\left(cl_p(A)\right) \cap [cl^*\left(cl_p(A)\right)]^c = \phi$. That is $F = \phi$, a contradiction. Thus, $cl^*\left(cl_p(A)\right) - A$ contain no non-empty $\tau^* - closed$ set in X. Conversely, assume that $cl^*\left(cl_p(A)\right) - A$ contains no non-empty $\tau^* - closed$ set. Let $A \subseteq G$, G is $\tau^* - open$. Suppose that $cl^*\left(cl_p(A)\right)$ is not contained in G. then $cl^*\left(cl_p(A)\right) \cap G^c$ is a non-empty $\tau^* - closed$ set of $cl^*\left(cl_p(A)\right) - A$ which is a contradiction. Therefore, $cl^*\left(cl_p(A)\right) - A \subseteq G$ and lence A is $\tau^* - gp - closed$.

Corollary: 3.12
A subset A of X is $\tau^* - gp - closed$ if and only $cl^*\left(cl_p(A)\right) - A$ contains no non-empty closed set in X.

Proof:
The proof follows from the theorem 3.11 and the fact that every closed set is $\tau^* - closed$ set in X.

Corollary: 3.13
A subset A of X is $\tau^* - gp - closed$ if and only if $cl^*\left(cl_p(A)\right) - A$ contains no non-empty open set in X.

Proof:
The proof follows from the theorem 3.11 and the fact that every open set is $\tau^* - open$ set in X.

Theorem: 3.14
If a subset A of X is $\tau^* - gp - closed$ and $A \subseteq B \subseteq cl^*\left(cl_p(A)\right)$, then B is $\tau^* - gp - closed$ set in X.

Proof:
Let A be a $\tau^* - gp - closed$ set such that $A \subseteq B \subseteq cl^*\left(cl_p(A)\right)$. Let U be a $\tau^* - open$ set of X such that $B \subseteq U$. Since A is $\tau^* - gp - closed$, we have $cl^*\left(cl_p(A)\right) \subseteq U$.

Now,

$$cl^*\left(cl_p(A)\right) \subseteq cl^*\left(cl_p(B)\right) \subseteq cl^*\left(cl^*(cl_p(A))\right) = cl^*\left(cl_p(A)\right) \subseteq U$$

That is $cl^*\left(cl_p(B)\right) \subseteq U$, U is $\tau^* - open$.

Therefore B is $\tau^* - gp - closed$ set in X.

The converse of the above theorem need not be true as seen from the following example.

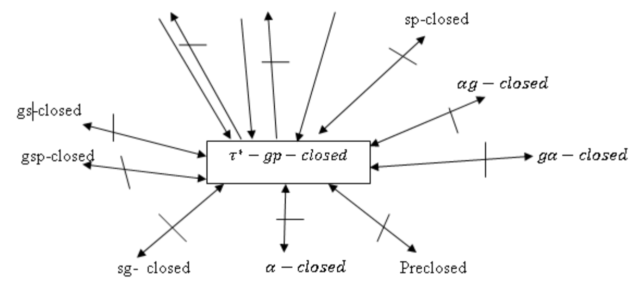
Example: 3.15
Consider the topological space $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. Let $A = \{c\}$ and $B = \{b, c\}$. Then A and B are $\tau^* - gp - closed$ sets in (X, τ). But $A \subseteq B$ is not a subset of $cl^*\left(cl_p(A)\right)$.

Theorem: 3.16
Let A be a $\tau^* - gp - closed$ in (X, τ). Then A is $g - closed$ if and only if $cl^*\left(cl_p(A)\right) - A$ is $\tau^* - open$.

Proof:
Suppose A is g-closed in X. Then, $cl^*\left(cl_p(A)\right) = A$ and so $cl^*\left(cl_p(A)\right) - A = \phi$ which is $\tau^* - open$ in X. Conversely, suppose $cl^*\left(cl_p(A)\right) - A$ is $\tau^* - open$ in X. Since A is $\tau^* - gp - closed$, by the theorem 3.11, $cl^*\left(cl_p(A)\right) - A$ contains no non-empty $\tau^* - closed$ set in X. Then, $cl^*\left(cl_p(A)\right) - A = \phi$. Hence, A is g-closed.

Remark 3.17
From the above discussion, we obtain the following implications.

Closed g-closed $\tau^* - closed$



A ⟶ B means A implies B, A ⟶̸ B means A does not imply B and A ⟷̸ B means A and B are independent.

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