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Bending Analysis Of Simply Supported And Clamped Circular Plate

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***Abstract* - The aim of study is static bending analysis of an isotropic circular plate using analytical, experimental method i.e. Classical Plate Theory, Finite Element software ANSYS And Universal Testing Machine (UTM). Circular plate analysis is done in cylindrical coordinate system by using Classical Plate Theory. The axisymmetric bending of circular plate is considered in the present study. The diameter of circular plate, material properties like modulus of elasticity (E), poissons ratio (µ) and intensity of loading is assumed at the initial stage study. In this research work, the effect of varying thickness of the plate on its deflection and bending stress is studied. So, the key point of research work is thickness variation of plate. Both simply supported and clamped boundary conditions subjected to uniformly distributed load and center concentrated / point load have been considered in the present study. Modelling and analysis of circular plate is done by FEA software ANSYS APDL. And the actual experiment perform on the UTM. Once deflection is obtained by using CPT, bending moments and bending stresses are easily calculated by usual relations. Analytical results of CPT are validated with ANSYS and UTM results.**

***Keywords*— isotropic circular plate, Classical Plate Theory, Finite Element software ANSYS, Universal Testing Machine.**

I. INTRODUCTION

Plates are extensively used in many engineering applications like roof and floor of building, deck slab of bridge, foundation of footing, water tanks, turbine disks etc. Plates used in such applications are normally subjected to lateral loads causing bending of the plate. Hence bending analysis of plate is of utmost importance. The geometry of the plate normally defined by middle plane which is plane equidistance from the top and bottom faces of the plate. The flexural properties of plate largely depends on its thickness rather than its two dimensions (length and width). The beam and plate theories are the active areas of research since the historical time. The classical engineering theory of bending due to Bernoulli and Euler dates back to1705 and had its origin in the first mathematical model of nature of the resistance of a beam developed by Galileo Galilei in 1638.The classical theory of plate bending had its origin in the pioneering work of Sophie Germain carried out in 1815. The theory reached maturity due to the well-known Kirchhoff hypothesis and the resolution of famous boundary conditions paradox by Kirchhoff in 1850. Bending of plates or plate bending refers to the deflection of a plate perpendicular to the plane of the plate under the action of external forces and moments. The amount of deflection can be determined by solving the differential equations of an appropriate plate theory. In this paper classical plate theory is used for bending analysis of isotropic circular plate. Analytical results obtained from CPT and ANSYS result are validate with the results of Experimental setup (UTM).This procedure are perform on two different material M.S. and Al .And also compare this result.

II. METHODOLOGY

Static bending analysis of an isotropic circular plate is done by analytical method ,FEA software ANSYS and Experimental setup UTM . Following are the four different cases considered for present study:-

1) Simply supported circular plate subjected to uniformly distributed load.
2) Clamped circular plate subjected to uniformly distributed load.
3) Simply supported circular plate subjected to centre concentrated or point load.
4) Clamped circular plate subjected to centre concentrated or point load.

At the initial stage of work, following standard values of constant terms are assumed:-

Plate diameter = 200mm
Poisson's ratio of steel (µ) = 0.3(M.S) and 0.33(Al)
Young's modulus of steel (E) = 200 GPa(M.S) and 70 GPa(Al)
Distributed load = 275 kPa.
Centre concentrated load = 275 kN

Thickness of the circular plate is varied throughout for various boundary and loading conditions as like 10mm, 20mm and 30mm.Then the flexural parameters like deflections and bending stresses are calculated for various thicknesses of the plate.

2.1 Classical Plate Theory

Classical Plate Theory is the thin plate theory based on Love-Kirchhoff's hypothesis which makes assumptions similar to those made by the Bernoulli-Navier hypothesis used in the theory of thin or shallow beams. It is also called as small deflection theory.

The following fundamental assumptions are made in the classical small deflection theory of thin homogenous elastic plates.

1. Straight line initially normal to the middle surface to the plate remains straight and normal to the deformed middle surface of the plate and unchanged in length.
2. Displacement w is assumed to be very small. This means the slope of the deflected surface is small and hence square of the slope would be negligible in comparison with unity.
3. The normal stresses x □□and y □□in plane shear stress xy □□are assumed to be zero at middle surface of the plate. i.e. □w □□□h□□.
4. Stress z □□i.e. transverse normal stress is small as compared to other stress components and may be neglected in stress strain relationship.□□, , □□z x y xy □□□□□□□□□□
5. The mid plane remains unstrained after bending.

The above assumptions, known as Kirchhoff's hypothesis, reduces the three dimensional plate problem to two dimensions. Hence only normal stresses and shear stresses would exists in the plate. In cylindrical coordinate system these stresses are the functions of r and □.

2.2 ANSYS Solution

Thin circular plate for various boundary conditions and loading conditions is analyzed by using ANSYSAPDL.ANSYS is one of the finite element analysis based software which is numerical technique based.The plate geometry is model in ANSYS APDL. Shell 181 4 node element is used for circular plateanalysis. This element is 2D element with six degrees of freedom. Deflections and bending stresses are calculated for various cases considered for present research work. Here meshing is done by mapped mesh with quad element.

2.3 UNIVERSAL TESTING MACHINE

A universal testing machine is used to test the tensile stress and compressive strength of materials. It is named after the fact that it can perform many standard tensile and compression tests on materials, components, and structures. Bend Test: This is a compression test where you support a length of material by spanning it across two supports on each end. There is nothing supporting the middle portion underneath of it. Then you press down from above directly in the middle of the span of material until the supported material breaks or reaches a specific distance. This test measures how strong the material in flexure (flexural strength) and how stiff it is (flexural modulus).

III. ILLUSTRATIVE EXAMPLE

A) Find out deflection and bending stresses in the simply supported (M.S) circular plate subjected to uniformly distributed load for following data.

Plate Diameter = 200mm

Plate Thickness= 10mm = 0.010m

Poisson’s ratio of steel (μ)= 0.3

Young’s modulus of steel (E) = 200 GPa

Distributed Load = 275 kPa.

Solution:

From a given data, radius of circular plate = 100 mm=0.1 m

For simply supported circular plate subjected to uniformly distributed load , flexural rigidity is given by following formula

$$D=\frac{Eh^3}{12\left(1-\mu^2\right)}=\frac{200\times10^9\times\left(0.010\right)^3}{12\left(1-0.3^2\right)}$$

= 18315.02

Deflection of a simply supported circular plate is given as follows

Deflection (w) $_{\max}$ = $\dfrac{qa^4}{64D}\dfrac{(5+\mu)}{(1+\mu)}=\dfrac{275\times10^3\times\left(0.1\right)^4\times5.3}{64\times18315.02\times1.3}$

= 9.56 x10^{-5}m =0.0956 mm.

The bending moments calculated by following formula

$$\left(M_r\right)_{\max}=\frac{q}{16}\left[\left(3+\mu\right)\left(a^2-r^2\right)\right]$$

The maximum bending moment is at r = 0. Hence M$_r$ becomes

$$=\frac{275\times10^3}{16}\left(3.3\times0.1^2\right)$$ = 567.187N-m = 567187N-mm

Similarly,

$$\left(M_\theta\right)_{\max}=-\frac{q}{16}\left[\left(1+3\mu\right)r^2-a^2\left(3+\mu\right)\right]$$ = 567187N-mm

The bending stresses in the circular plate are found out from following equations

$$\left(\sigma_r\right)_{\max}=\frac{3}{8}q\eta^2\left(3+\mu\right)$$

$$=\frac{3}{8}\times275\times10^3\times\left(\frac{0.1}{0.010}\right)^2\times3.3$$ = 34031250 N/m^2

= 34.03 N/mm^2 or MPa

$$\left(\sigma_\theta\right)_{\max}=\frac{3}{8}q\eta^2\left(3+\mu\right)$$ = 34.03 MPa

Therefore maximum stresses in r and θ are same.

$$\left(\sigma_r\right)_{\max}=\left(\sigma_\theta\right)_{\max}$$ = 34.03 MPa.

B) Following is the ANSYS solution obtained for simply supported circular plate of 10mm thickness with UDL:-

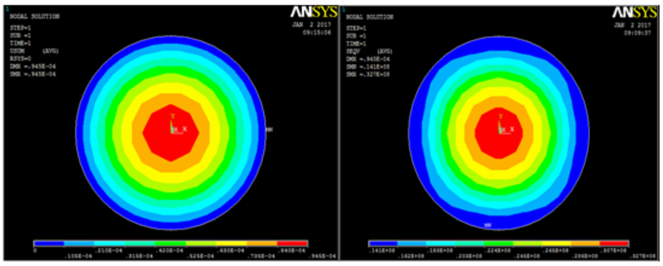
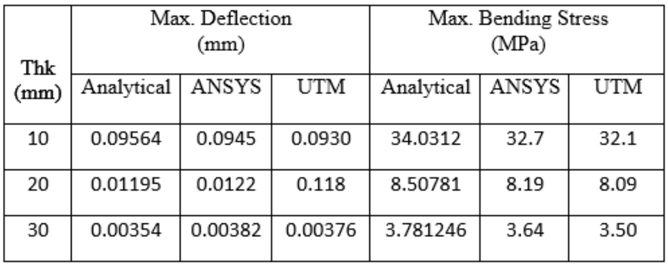


Figure1,2 show Deflection and bending stress contour for 10 mm thickness of simply
supported plate with UDL of 275kPa

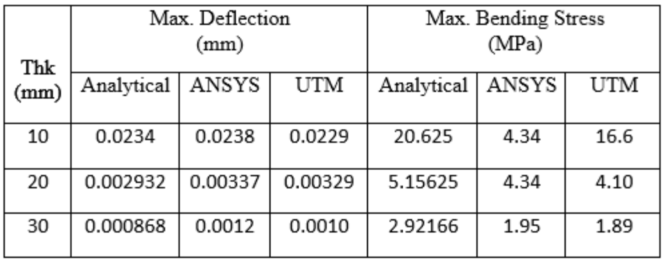
Results

Table-1 Comparison of deflection and bending stress for simply supported (M.S) circular plate subjected to UDL obtained from ANALYTICAL, ANSYS and UTM



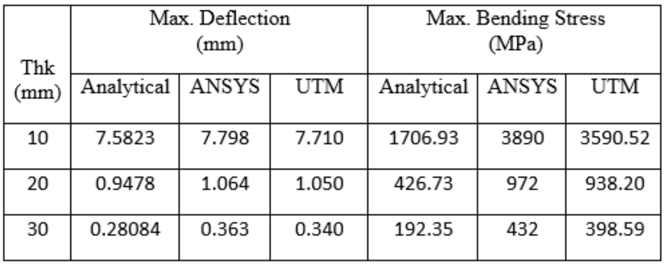
Thk (mm)	Max. Deflection (mm)			Max. Bending Stress (MPa)		
	Analytical	ANSYS	UTM	Analytical	ANSYS	UTM
10	0.09564	0.0945	0.0930	34.0312	32.7	32.1
20	0.01195	0.0122	0.118	8.50781	8.19	8.09
30	0.00354	0.00382	0.00376	3.781246	3.64	3.50

Table-2 Comparison of deflection and bending stress for clamed supported (M.S) circular plate subjected to UDL obtained from ANALYTICAL, ANSYS and UTM



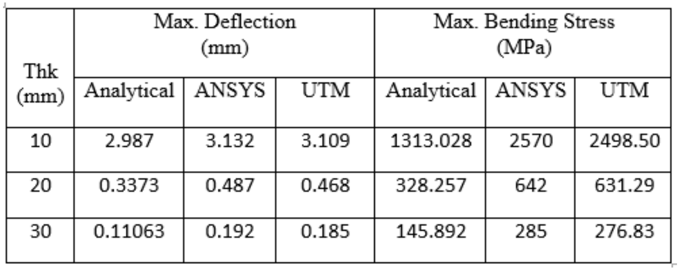
Thk (mm)	Max. Deflection (mm)			Max. Bending Stress (MPa)		
	Analytical	ANSYS	UTM	Analytical	ANSYS	UTM
10	0.0234	0.0238	0.0229	20.625	4.34	16.6
20	0.002932	0.00337	0.00329	5.15625	4.34	4.10
30	0.000868	0.0012	0.0010	2.92166	1.95	1.89

Table-3 Comparison of deflection and bending stress for simply supported (M.S) circular plate subjected to point load obtained from ANALYTICAL, ANSYS and UTM



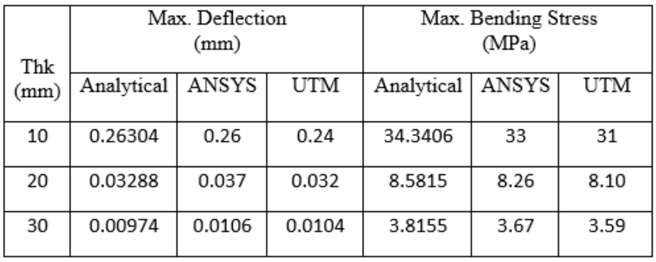
Thk (mm)	Max. Deflection (mm)			Max. Bending Stress (MPa)		
	Analytical	ANSYS	UTM	Analytical	ANSYS	UTM
10	7.5823	7.798	7.710	1706.93	3890	3590.52
20	0.9478	1.064	1.050	426.73	972	938.20
30	0.28084	0.363	0.340	192.35	432	398.59

Table-4 Comparison of deflection and bending stress for clamed supported (M.S) circular plate subjected to point load obtained from ANALYTICAL, ANSYS and UTM



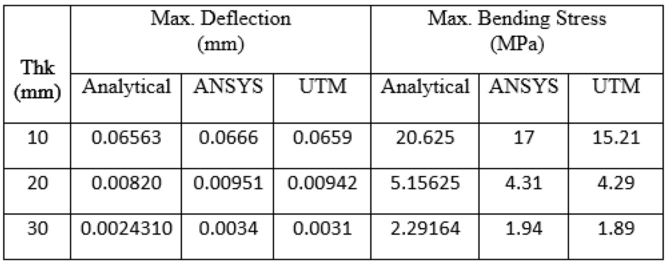
Thk (mm)	Max. Deflection (mm)			Max. Bending Stress (MPa)		
	Analytical	ANSYS	UTM	Analytical	ANSYS	UTM
10	2.987	3.132	3.109	1313.028	2570	2498.50
20	0.3373	0.487	0.468	328.257	642	631.29
30	0.11063	0.192	0.185	145.892	285	276.83

Table-5 Comparison of deflection and bending stress for simply supported (Al) circular plate subjected to UDL obtained from ANALYTICAL, ANSYS and UTM



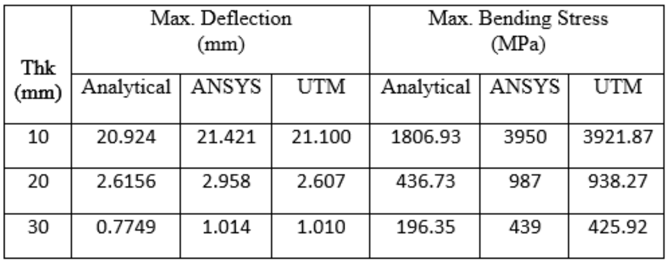
Thk (mm)	Max. Deflection (mm)			Max. Bending Stress (MPa)		
	Analytical	ANSYS	UTM	Analytical	ANSYS	UTM
10	0.26304	0.26	0.24	34.3406	33	31
20	0.03288	0.037	0.032	8.5815	8.26	8.10
30	0.00974	0.0106	0.0104	3.8155	3.67	3.59

Table-6 Comparison of deflection and bending stress for clamed supported (Al) circular plate subjected to UDL obtained from ANALYTICAL, ANSYS and UTM



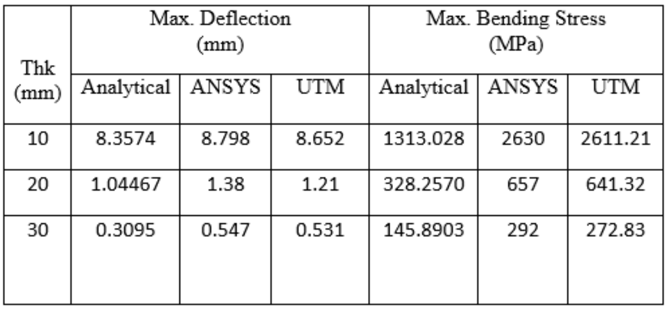
Thk (mm)	Max. Deflection (mm)			Max. Bending Stress (MPa)		
	Analytical	ANSYS	UTM	Analytical	ANSYS	UTM
10	0.06563	0.0666	0.0659	20.625	17	15.21
20	0.00820	0.00951	0.00942	5.15625	4.31	4.29
30	0.0024310	0.0034	0.0031	2.29164	1.94	1.89

Table-7 Comparison of deflection and bending stress for simply supported (Al) circular plate subjected to point load obtained from ANALYTICAL, ANSYS and UTM



Thk (mm)	Max. Deflection (mm)			Max. Bending Stress (MPa)		
	Analytical	ANSYS	UTM	Analytical	ANSYS	UTM
10	20.924	21.421	21.100	1806.93	3950	3921.87
20	2.6156	2.958	2.607	436.73	987	938.27
30	0.7749	1.014	1.010	196.35	439	425.92

Table-8 Comparison of deflection and bending stress for clamed supported (Al) circular plate subjected to point load obtained from ANALYTICAL, ANSYS and UTM



Thk (mm)	Max. Deflection (mm)			Max. Bending Stress (MPa)		
	Analytical	ANSYS	UTM	Analytical	ANSYS	UTM
10	8.3574	8.798	8.652	1313.028	2630	2611.21
20	1.04467	1.38	1.21	328.2570	657	641.32
30	0.3095	0.547	0.531	145.8903	292	272.83

Discussion

Table 1 shows comparison of deflection and bending stress obtained from analytical method, ANSYS and UTM for simply supported (M.S) circular plate subjected to UDL. From Table 1, results obtained for deflections and bending stresses from analytical method gives good agreement with the results obtained from ANSYS and UTM Results obtained from both the methods matches nearly about 95 to 97%. Table 2 shows comparison of deflections and bending stresses obtained from analytical method, ANSYS and UTM for clamped circular (M.S) plate subjected to UDL. From Table 2 results obtained from analytical equations, ANSYS and UTM for deflection and bending stress for clamped plate with UDL nearly matches about 85 to 90%.Table 3 and Table 4 shows comparison of deflection and bending stress for simply supported and clamped circular (M.S) plate with centre concentrated load or point load respectively. Deflection obtained from analytical method, ANSYS and UTM for simply supported and clamped circular plate with centre concentrated load are approximately equal for lower value of thickness, results obtained for bending stress from analytical method, ANSYS and UTM for simply supported and clamped circular plate with centre concentrated load matches nearly about 50%. Table 5,6,7 and 8 shows comparison of deflection and bending stress for simply supported and clamped circular (Al) plate with UDL and point load respectively. Deflection obtained from analytical method, ANSYS and UTM for simply supported and clamped circular plate with centre concentrated load and UDL are approximately equal for lower value of thickness, results obtained for bending stress from analytical method, ANSYS and UTM for simply supported and clamped circular plate with centre concentrated load and UDL matches nearly about 50%.

CONCLUSIONS

From the parametric study of bending analysis of simply supported and clamped isotropic circular plate by using Analytical method , ANSYS and UTM following conclusions are drawn:

1) Deflection and bending stress in thin isotropic circular plate decreases with increase in thickness. This is so because; the plate is thin and isotropic where shear deformation is not considered. If the plate is thick then the effect of shear deformation is more pronounced and the results obtained may vary.

2) Flexural parameters obtained from analytical method and ANSYS shows good agreement of results with each other only for thin simply supported and clamped circular plate subjected to UDL.

3) For simply supported and clamped circular plate with UDL, bending stresses in radial and circumferential direction are equal while it is not equal in the case of concentrated or point loads due to concentration of load or stress at a specific point.

4) Results for deflection and bending stress obtained from analytical equations does not gives good agreement with the results obtained from ANSYS and UTM for both simply supported and clamped circular plate subjected to centre concentrated load or point load because of effect of stress concentration due to application of concentrated or point load.

5) Bending stresses in radial as well as circumferential direction increases with decrease in thickness. This is so because the plate is thin where shear effect is not considered. On the other hand, if the plate is thick then the effect of shear deformation is more pronounced.

REFERENCES

[1] S. Timoshenko and S. Woinowsky-Krieger, “Theory of plates and shells”, 2nd Edition, Tata McGraw Hill Education Private Limited, 2010.

[2] G. C. Mekalke, M. V. Kavade and S. S. Deshpande, “Analysis of a plate with a circular hole by FEM”, IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE) ISSN: 2278-1684, pp.25-30, 2012.

[3] B.C.L.Vanam, M. Rajyalakshmi and R. Inala,“Static analysis of an isotropic rectangular plate using finite element analysis(FEA)”, Journal of Mechanical Engineering Research, Vol. 4(4), pp.148-162, April 2012.

[4] Wang Yun, Xu Rongqiao and Ding Haojiang,“Three-dimensional solution of axisymmetric bending of functionally graded circular plates”, Elsevier Composite Structures 92, pp.1683–1693, 2010.

[5] A. F. Kirstein and R. M. Woolley, “Symmetrical bending of thin circular elastic plates on equally spaced point support”, Journal of Research of the National Bureau of Standards - C. Engineering and Instrumentation, Vol. 71 C, No.1,1967.

[6] Biswa Prakash Sukla,“The FEM analysis on circular stiffened plates using ANSYS”, Bachelor of Technology diss.,National Institute of Technology, Rourkela (India), 2009.

[7] N. L. Khobragade and K. C. Deshmukh,“Thermal deformation in a thin circular plate due to a partially distributed heat supply",Sadhana Vol. 30, Part 4, pp. 555–563, 2005.

[8] H. M. Montrey,“Bending of a circular sandwich plate by load applied through an insert”,U.S.D.A. Forest Service Research Paper FPL 201, 1973.

[9] H. D. Conway and K. A. Farnham, “Deflection of uniformly loaded circular plate with combinations of clamped, simply supported and free boundary conditions”, Int. J. Mech. Sci. Pergamon Press Ltd. Vol. 9, pp. 661-671, 1967.

[10] Murat Altekin and R Faruk Yükseler,“Large deflection analysis of clamped circular plates”, Proceedings of the World Congress on Engineering, Vol. III, London, U.K., 2011.