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TWO NON-IDENTICAL UNIT SYSTEM WITH FAILURES DUE TO MECHANICAL FAULT AND RANDOM SHOCKS

Dr. Sushant Pandey
Department of Management,
Ishan Institute Of Management & Technology
Greater Noida

***Abstract*— The present paper deals with the analysis of a two unit redundant system in which both the units are dissimilar. Here the units can fail due to mechanical fault as well as due to random shocks.**

Using regenerative point technique with Markov renewal process, various reliability characteristics of interest are obtained..

***Index Terms*— Random Shocks, Sojourn Time.**

I. INTRODUCTION

Several authors including [1,4,5] working in the field of reliability have analysed many engineering systems with the assumption that all the units of the system are of similar type. But there exists some two unit standby systems in which both the units are dissimilar with different costs and operating conditions.

Keeping the above view, we in this chapter analysed a two unit redundant system in which both the units are dissimilar. Here the units can fail due to mechanical fault as well as due to random shocks.

Using regenerative point technique with Markov renewal process, the following reliability characteristics of interest are obtained.

(1) Transition and steady state transition probabilities
(2) Mean Sojourn times in various states
(3) Mean time to system failure (MTSF)

II. MODEL DESCRIPTION AND ASSUMPTIONS

A. The system consists of only two non-identical units in which first is operative and the second unit is kept as cold standby.

B. First unit can sustain almost two shocks i.e. if the unit does not fail in first shock then it will definitely fail in the second shock. Also it can fail directly due to mechanical fault.

C. Second unit can fail due to mechanical fault as well as due to random shocks and it cannot sustain more than one random shock.

D. A single repair facility is considered in the system.

E. Second unit is repairable if it is failed due to mechanical fault otherwise in case of random shocks send it for replacement.

F. The priority in repair and replacement is given to the second unit over first unit.

All the failure time distributions are assumed to be negative exponential while the distribution of repair and replacement time are arbitrary.

III. NOTATION AND SYMBOLS

N_O : Normal unit kept as operative
N_S : Normal unit kept as cold standby
N_{O1} : Normal unit as operative after observing first random shock
F_r : Failed unit under repair
F_{wr} : Failed unit waiting for repair
F_{rep} : Failed unit under replacement
α : Constant rate of occurring first random shock to the first unit
β : Constant rate of occurring second random shock to the first unit
γ : Constant failure rate of first unit failed due to mechanical fault

δ : Constant failure rate of second unit failed due to random shock
λ : Constant failure rate of second unit failed due to mechanical fault
f(.), F(.) : pdf and cdf of time to repair of first unit
g(.), G(.) : pdf and cdf of time to replacement of second unit failed due to random shock
h(.), H(.) : pdf and cdf of time to repair of second unit failed due to mechanical fault

Using the above notation and symbols the possible states of the system are

<u>Up States</u>

$S_0 \equiv (N_O, N_S)$ $S_1 \equiv (N_{O1}, N_S)$
$S_2 \equiv (F_r, N_O)$

<u>Down States</u>

$S_3 \equiv (F_{wr}, F_{rep})$ $S_4 \equiv (F_{wr}, F_r)$

The possible transitions between the states for analysis the above model are

$S_0 \to S_1, S_0 \to S_2, S_1 \to S_2, S_2 \to S_0, S_2 \to S_3, S_2 \to S_4, S_3 \to S_2$

$S_4 \to S_2$

IV. TRANSITION PROBABILITIES

Let T_0 (=0), $T_1, T_2, ...$ be the epochs at which the system enters the states $S_i \in E$. Let X_n denotes the state entered at epoch T_{n+1} i.e. just after the transition of T_n. Then $\{T_n, X_n\}$ constitutes a Markov-renewal process with state space E and

$$Q_{ik}(t) = \Pr[X_{n+1} = S_k, T_{n+1} - T_n \le t \mid X_n = S_i] \quad ....(1)$$

is semi Markov-Kernal over E. The stochastic matrix of the embedded Markov chain is

$$P = p_{ik} = \lim_{t\to\infty} Q_{ik}(t) = Q(\infty) \quad ....(2)$$

Thus,we get

$$p_{01} = \frac{\alpha}{\alpha+\gamma}$$

$$p_{02} = \frac{\gamma}{\alpha+\gamma}$$

$$p_{12} = 1$$

$$p_{20} = f^*(\delta+\lambda)$$

$$p_{23} = \frac{\delta}{\delta+\lambda}\,[1 - f^*(\delta+\lambda)]$$

$$p_{23} = \frac{\lambda}{\delta+\lambda}\,[1 - f^*(\delta+\lambda)]$$

$$p_{32} = 1$$

$$p_{42} = 1 \quad ….(3\text{-}10)$$

From the above probabilities the following relations can be easily verified as;

$$p_{01} + p_{02} = 1 = p_{12} = p_{32} = p_{42}$$
$$p_{20} + p_{23} + p_{24} = 1 \quad ….(11\text{-}12)$$

V. MEAN SOJOURN TIMES

The mean time taken by the system in a particular state S_i before transiting to any other state is known as mean sojourn time and is defined as

$$\mu_i = {}_0\int^\infty P[T>t]\,dt \quad ....(13)$$

Where T is the time of stay in state S_i by the system.

To calculate mean sojourn time μ_I in state S_i, we assume that so long as the system is in state S_i, it will not transit to any other state. Therefore;

$$\mu_0 = {}_0\int^\infty e^{-(\alpha+\gamma)t}\,dt = \frac{1}{\alpha+\gamma}$$

$$\mu_1 = {}_0\int^\infty e^{-\beta t}\,dt = \frac{1}{\beta}$$

$$\mu_2 = {}_0\int^\infty e^{-(\delta+\lambda)t}\,\overline{F}(t)dt = \frac{1}{\delta+\lambda}\,[1 - f^*(\delta+\lambda)]$$

$$\mu_3 = {}_0\int^\infty \overline{G}(t)\,dt = {}_0\int^\infty t.g(t)\,dt$$

$$\mu_4 = {}_0\int^\infty \overline{H}(t)dt = {}_0\int^\infty t.h(t)\,dt \quad ….(14\text{-}18)$$

<u>Contribution to Mean Sojourn Time</u>

For the contribution to mean sojourn time in state $S_i \in E$ and non-regenerative state occurs, before transiting to $S_j \in E$, i.e.,

$$m_{ij} = -\int t.q_{ij}(t)\,dt = -q'^*_{ij}(0) \quad ….(19)$$

Therefore,

$$m_{01} = {}_0\int^\infty \alpha.t.e^{-(\alpha+\gamma)t}\,dt = \frac{\alpha}{(\alpha+\gamma)^2}$$

$m_{02} = {}_0\int^{\infty} \gamma.t.e^{-(\alpha+\gamma)t}\, dt = \dfrac{\gamma}{(\alpha+\gamma)^2}$

$m_{12} = {}_0\int^{\infty} \beta.t.e^{-\beta t}\, dt = \dfrac{1}{\beta}$

$m_{20} = {}_0\int^{\infty} t.e^{-(\delta+\lambda)t} f(t)\, dt$

$m_{23} = \delta.{}_0\int^{\infty} t.e^{-(\delta+\lambda)t}\, \overline{F}(t)\, dt$

$m_{24} = \lambda.{}_0\int^{\infty} t.e^{-(\delta+\lambda)t}\, \overline{F}(t)\, dt$

$m_{32} = {}_0\int^{\infty} t.g(t)\, dt$

$m_{42} = {}_0\int^{\infty} t.h(t)\, dt$
….(20-27)

Hence

$m_{01} + m_{02} = \dfrac{1}{\alpha+\gamma} = \mu_0$

$m_{12} = \dfrac{1}{\beta} = \mu_1$

$m_{20} + m_{23} + m_{24} = \dfrac{1}{\delta+\lambda}[1 - f^*(\delta+\lambda)] = \mu_2$

$m_{32} = {}_0\int^{\infty} t.g(t)\, dt = \mu_3$

$m_{42} = {}_0\int^{\infty} t.h(t)\, dt = \mu_4$ ….(28-32)

VI. MEAN TIME TO SYSTEM FAILURE (MTSF)

To obtain the distribution function $\pi_i(t)$ of the time to system failure with starting state S_0.

$\pi_0(t) = Q_{01}(t)\$\pi_1(t) + Q_{02}(t)\$\pi_2(t)$

$\pi_1(t) = Q_{12}(t)\$\pi_2(t)$

$\pi_2(t) = Q_{20}(t)\$\pi_0(t) + Q_{23}(t) + Q_{24}(t)$ ...(.33-35)

Taking Laplace Stieltjes transform of relations (33-35) and solving them for $\tilde{\pi}_0(s)$ by omitting the argument ‘s’ for brevity,one gets

$\tilde{\pi}_0(s) = N_1(s)/ D_1(s)$ ...(36)

where

$N_1(s) = \tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{23} + \tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{24} + \tilde{Q}_{02}\tilde{Q}_{23} + \tilde{Q}_{02}\tilde{Q}_{24}$ ....(37)

and

$D_1(s) = 1 - \tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{20} - \tilde{Q}_{02}\tilde{Q}_{20}$ ….(38)

The mean time to system failure (MTSF) when the system starts from S_0 is

$E(T) = -\dfrac{d}{ds}\pi_0(s)|_{s=0} = \dfrac{D'_1(0) - N'_1(0)}{D_1(0)}$ ….(39)

where

$N_1 = \mu_0 + \mu_1 p_{01} + \mu_2$ ….(40)

and

$D_1 = 1 - p_{20}$ ….(41)

REFERENCES

[1] Agnihotri, R.K, Singhal, Govind and Khandelwal, S.K. (1992) : “Stochastic analysis of a two unit redundant system with two types of failure”, Microlectron, Reliab. 32, 901-904

[2] Chiang, J.H. and Yuan, J. (2001) : Optimal maintenance policy for a Markovian system under periodic inspection, Reliab. Engg. Syst. Safety, Vol. 71(2), pp. 165-172.

[3] Chin, C.T., Chen, Y.W. and Yuan, J. (2003) : On a dynamic preventive maintenance policy for a system under inspection, Reliab. Engg. Syst. Safety, Vol. 80(1), pp. 41-47.

[4] Goel, L.R., and Gupta, P. (1984) : “Analysis of a two engine aero plane model with two types of failure and preventive maintenance”, Microelectron Reliab., 24, 663-666

[5] Goyal, V. and Murari, K. (1984) : ” Cost analysis in a two unit cold standby system with regular repairman and patience time”, Microelectron Reliab., 24, 453-459

[6] Moustafa. M.S. (1997) : “Reliability analysis of k-out-of-n :G systems with dependent failures and imperfect coverage”, Reliab. Eng. and Sys. Safety, Vol. 58, 15-57.