

THE KUYPERS EFFECT: ANGULAR-MOMENTUM CONSERVATION IMPLIES GLOBAL c IN GRAVITY

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Abstract- A young astronomer's by now ten years old results are re-told and put in perspective. The implications are far-reaching. Angular-momentum shows its clout not only in quantum mechanics where this is well known, but is also a major player in the space-time theory of the equivalence principle and its ramifications. In general relativity, its fundamental role was largely neglected for the better part of a century. A children's device – a friction-free rotating bicycle wheel suspended from its hub that can be lowered and pulled up reversibly – serves as an eye-opener. The consequences are embarrassingly far-reaching in reviving Einstein's original dream.

Key words: Bicycle wheel, angular momentum conservation, equivalence principle, gravitation, quantum mechanics, global validity of c , black-hole theory, experimental safety. (January 27, 2015)

I. INTRODUCTION

Angular momentum is a fundamental conserved quantity in nature like energy. It is in the simplest case defined by "rotation frequency times mass times squared radius." The most intuitive application is a bicycle wheel suspended from its hub which can be vertically lowered and pulled up again in a frictionless manner. The young astronomer Heinrich Kuypers showed in an abstract published in 2003 [1] and subsequently in a doctoral thesis (unpublished) that an unfamiliar consequence is implicit in angular-momentum conservation: a gravitational-redshift proportional size change [2].

II. A REVIEW OF THE RESULT

The new effect is straightforward to derive. If the mass of the frictionless bicycle wheel is for simplicity assumed to be condensed into the ideally infinitely thin outer rim, then the following simple textbook formula [3] suffices for a description of the conserved angular momentum L when the wheel is allowed to rotate frictionlessly at a constant – say horizontal – orientation in space:

$$L = \omega m r^2 = \text{const.} \quad (1)$$

In this easy-to-remember formula ("Lomrr" which almost sounds like "l'hombre"), ω is the rotation rate and m the mass and r the radius.

Next, look at Einstein's epoch-making discovery of gravitational redshift, or equivalently gravitational clock slowdown downstairs, which is a monotonic function of the height difference. It can be written as

$$\omega' = k \omega, \quad (2)$$

where $k < 1$ is the redshift factor relative to above. The constant k depends on the height difference as originally described by Einstein in the form

$$k = (1 + \Phi/c^2), \quad (3)$$

with Φ the (by definition negative) gravitational potential as a function of height assumed in the equivalence principle and c the speed of light [4].

Eq.(2) means that the lowered wheel, while keeping its original angular momentum L , cannot but rotate *more slowly* downstairs in gravity in dependence on the height difference. Its locally constant rotation rate makes the wheel qualify as a "clock." For example, the Schwarzschild metric [5], described nine years later as an implication of the full Einstein equation of 1916 [6], implies that k approaches zero at the surface ("horizon" in Rindler's terminology) of a black hole. But we can stick here to the simpler context of the equivalence principle proper without loss of generality. What does the validity of Eq.(2) mean for Eq.(1)?

If ω is changed into ω' downstairs by Eq.(2) in the conservation law of Eq.(1), it is clear that either m or r or both must undergo a *compensatory change* down there since L is conserved. At first sight, infinitely many possibilities open themselves up for an m' or an r' or both, in order to in combination with ω' keep the angular-momentum L of Eq.(1) constant.

However, serendipity allows that another physical fact of nature comes to the aid. It was discovered two decades later and makes the consequences that Eq.(2) entails for Eq.(1) well defined. This is the famous "creation and annihilation operators" of Dirac's quantum mechanics [7]. We saw already that *photons* that are emitted downstairs possess – in spite of their appearing normal locally – a reduced frequency relative to above by virtue of Eq.(2) [4]. The Dirac mechanism which allows particles to be interconverted into photons and vice versa, *constrains* the masses of elementary particles created downstairs. For example, an electron and a positron which both have the same mass can jointly get annihilated into two mutually opposite 511 keV gamma-ray photons, a mechanism that has medical applications ("PET scan") and works at all height levels on earth – despite the fact that the lower-level 511 keV photons have less energy (are redshifted) relative to those above. As a consequence of Dirac's result, *all masses* locally at rest downstairs are *reduced* compared to above by the relative gravitational redshift factor k of Eq.(2). This for some reason almost never mentioned fact, cf. [8], means for our wheel that its mass m' valid downstairs is

$$m' = k m. \quad (4)$$

Now that m' is fixed, so is r' : Inserting both Eq.(2) and Eq.(4) into Eq.(1) yields the following astounding result:

$$r' = r/k. \quad (5)$$

That is, *all* material objects at rest downstairs in gravitation are *linearly increased in size* by the gravitational redshift factor k valid relative to above [9]. And so is space itself.

III. IMPLICATIONS

Eq.(5) when taken at face value (which is allowed to do as we shall see in a moment) has an astounding consequence when combined with Eq.(2): the speed of light in the vacuum, c , proves to be an invariant across height levels. To see this, it suffices to start out with $\omega = 2\pi/T$ with T the rotation period of the wheel valid upstairs, and to then multiply it with the unit length r valid upstairs. In this way a constant linear velocity for the rim of the rotating wheel (a certain small fraction of c) is obtained. Second, one can do the same thing downstairs with the primed variables on the right hand side of Eq.(1). Here one starts out with $\omega' = 2\pi/T'$ with T' the rotation period valid downstairs, and multiplies with the unit length r' valid downstairs. In this way a constant linear velocity for the rim of the rotating wheel (a certain small fraction of c) is obtained again. In both cases it is the *same* fraction of c . Therefore, *both* time *and* space are concomitantly transformed in gravity such that

$$c = \text{globally constant.} \quad (6)$$

This result, implicit in Eqs.(2) and (5) combined, rehabilitates c as a *global constant* in the equivalence principle. Einstein's so reluctantly arrived-at opposite conclusion [4] is therefore no longer necessary and possible. The finding that validity of Eq.(6) is enforced by angular-momentum conservation in the equivalence principle is Kuypers' main result [1,2].

IV. A PROBLEM

The result of Eq.(6) obtained by Kuypers is maximally astounding. It not only upsets more than a century old wisdom, it also formally contradicts an indubitable result valid in special relativity and by implication in the equivalence principle: the fact that transversal distances are conserved in special relativity. The latter fact, sometimes called the "parallel railroad-tracks principle," reads:

$$r\text{-transversal}' = r\text{-transversal.} \quad (7)$$

Eq.(7) is bound to hold true in between the lowered wheel's radius $r\text{-transversal}'$ and the radius $r\text{-transversal}$ of the same wheel valid at the original higher-up position. It goes without saying that Eq.(7) formally contradicts Eq.(5) and hence also Eq.(6). Therefore, a logical impasse appears to have been reached at first sight.

V. THE SOLUTION

Fortunately, the contradiction arrived at is *not* a logical "contradictio in adjectu" because Eq.(7) is not a genuine identity but rather involves a projection effect: It is only "under vertical projection" that Eq.(7) holds true. That is to say: even though lateral sizes *do* map upon each other under vertical light rays in the equivalence principle by virtue of special relativity [4], this *projective constraint* means something new in light of Eq.(5).

It turns out that there is no contradiction. Both the conserved projection of Eq.(7) and the size increase of Eq.(5) are valid: Locally the objects *are* bigger downstairs, but they do *not look* so from above. How can one be sure that this is the solution? Answer: by letting the rotating wheel rotate not about a vertical axis as before, but rather about a horizontal axis. In this case, Eq.(5) is bound to remain manifestly valid for the *vertical wheel diameter* of the upright wheel, while simultaneously the *horizontal diameter* of the upright wheel is "observationally

compressed" by virtue of Eq.(7). Thus the wheel looks like a vertical ellipse under the influence of Eq.(5). This effect may some day become empirically observable on a neutron star where the ratio is almost 2:1.

VI. A PARALLEL CASE

If the described way out appears like a "last resort" to the reader, there exists a "direct analog" familiar from special relativity: the famous "garage paradox." The latter describes, not an optically masked *expansion* as is at stake here, but rather an optically masked *contraction*. Specifically, a fast-moving quadratic two-dimensional "automobile" is well known to momentarily fit into a garage that is shorter in its length by the Lorentz factor but has the width of the same car at rest. (We neglect the subsequent braking process inside the garage.) The point is that the Lorentz-contracted automobile remained isotropic – quadratic – in its own frame even though this fact is optically masked. Analogously here: the Kuypers-expanded upright bicycle wheel remains isotropic – circular – in its own frame. That is, Kuypers' "gravitational size expansion" and FitzGerald and Lorentz's "kinematic size contraction" represent *twin results* in special relativity. One may even speak of a duality.

This finishes the present account of Kuypers' finding.

VII. AN UNUSUAL PLEDGE

I herewith pledge for the acceptance-at-long-last of Kuypers' thesis [2] as a doctoral dissertation. Its impact appears comparable to that of Louis de Broglie's likewise at first unwelcome doctoral thesis. The impact of Kuypers' main result, Eq.(5), is equally seminal. It for example implies via the Schwarzschild metric [10] that the well-known infinite *temporal* distance from the outside world of the horizon of a black hole (at which k approaches zero) [11] is matched by an infinite *spatial* distance valid from the outside world.

A. Consequences of Equation Six, Part I

Since Eq.(6) is implicit in the Kuypers equation, Eq.(5), it transpires that many "formally allowed" transforms of the Einstein field equations get strongly constrained. Therefore Eq.(6) profoundly changes the properties of black holes. It implies that nothing can enter the horizon in finite outer time – not even by quantum tunneling – because the spatial distance has become *as infinite* from the outside world as the temporal distance has always been known to be [11]. Thus at least one theoretically accepted if empirically unconfirmed combined general-relativistic and quantum effect, the famous Hawking radiation [12], ceases to be physical in the wake of $c\text{-global}$. This could be seen as a rather arcane opposition between a young man of 1963 and a young man of 2005.

B. Consequences of Equation Six, Part II

There is a second clashing point between Kuypers' $c\text{-global}$ and the "establishment" if you so wish. It concerns the validity of the Friedmann solution to the Einstein equation of 1924 [12]. This is because a "speed of global expansion" can no longer be added to a "global c ." In light of this unfamiliar fact, a certain "gut reaction" to Kuypers' result (Eq.6) is clearly understandable.

This explains in retrospect why the third referee chosen by the faculty could single-handedly “kill” the promotion by refusing to give a grade to the dissertation. The two “very good” grades given by the supervisor and the maximally prestigious external co-referee did not warrant a further opinion, the faculty told me. This is understandable in light of the novelty of Eq.(6): A student cannot be allowed to upset a ruling paradigm.

C. Consequences of Equation Six, Part III

The collision with Hawking radiation implies that a prestigious terrestrial experiment – announced to be re-started at twice its former world-record energy in March 2015 – will be well advised to renew its now 7 years old safety report [13] *before* the re-start. For Kuypers’ thesis implies that the experiment relies on *false physics* (neglect of c -global). Note that this would not be the first time that false physics causes a catastrophe as the *Eniwetak* example shows. But this time around, the falsity is known beforehand. The proverb *ex falso quodlibet* (“from the false, anything [can follow]”) includes even existential risks.

VIII. A PERSONAL CONCLUSION

To conclude, I apologize to my former student that it took me ten years to rehabilitate him with the present paper. An old textbook formula – Eq.(1) – proves to possess this healing power. We all have to forgive the community for having overlooked Kuypers’ thesis for so long. I know for sure that the young generation – astronomer Kuypers works as an esteemed highschool teacher – will be particularly grateful to him: Can there be a simpler physical sentinel than Eq.(1)? It is a long time that “high-school mathematics” led to a major new insight in physics. This situation is bound to change in the wake of *l’hombre*. The ultimate reason for this is that physics is based on the “intrinsically simple” – or synonymously the “beautiful” as Dirac called it [14].

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